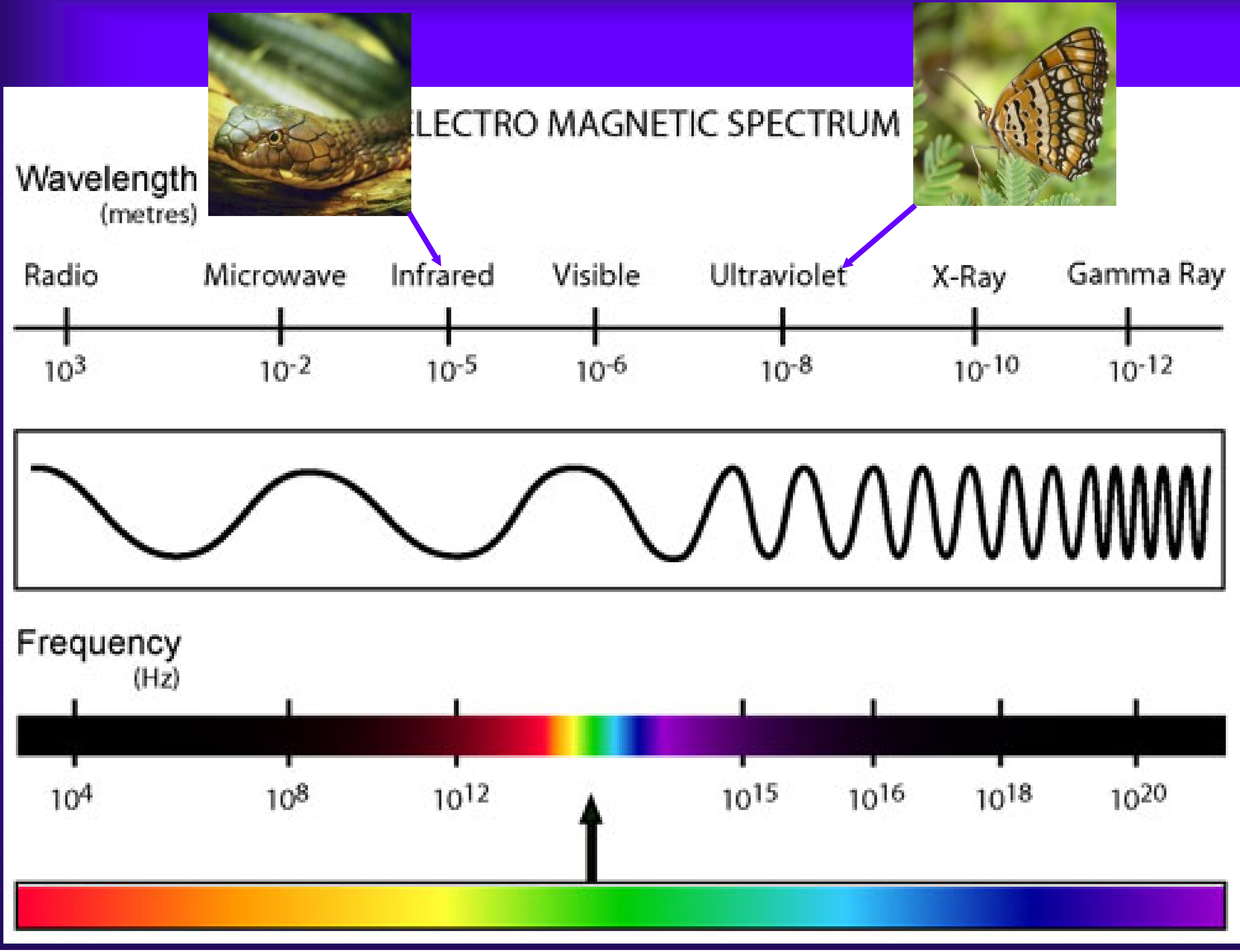
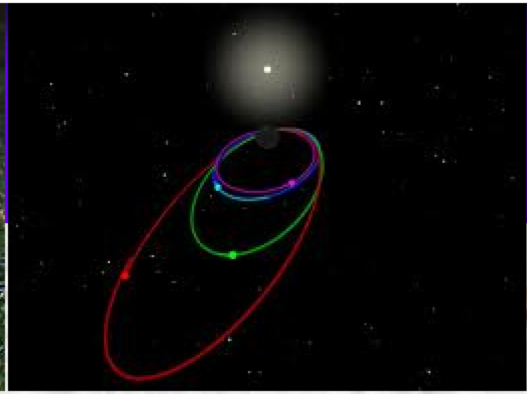
Electromagetic Wave

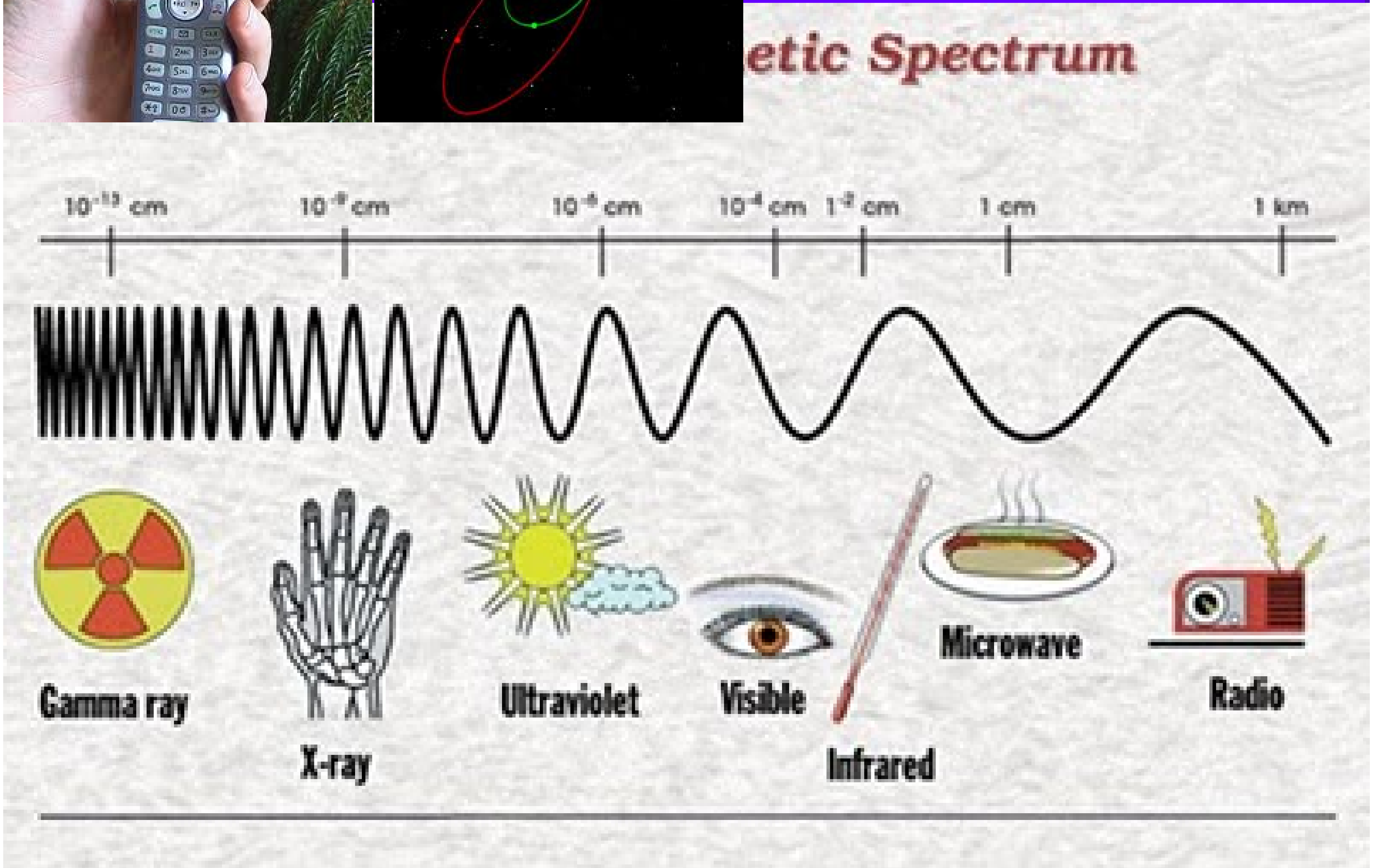
MAXWELL EQUATIONS

Contents

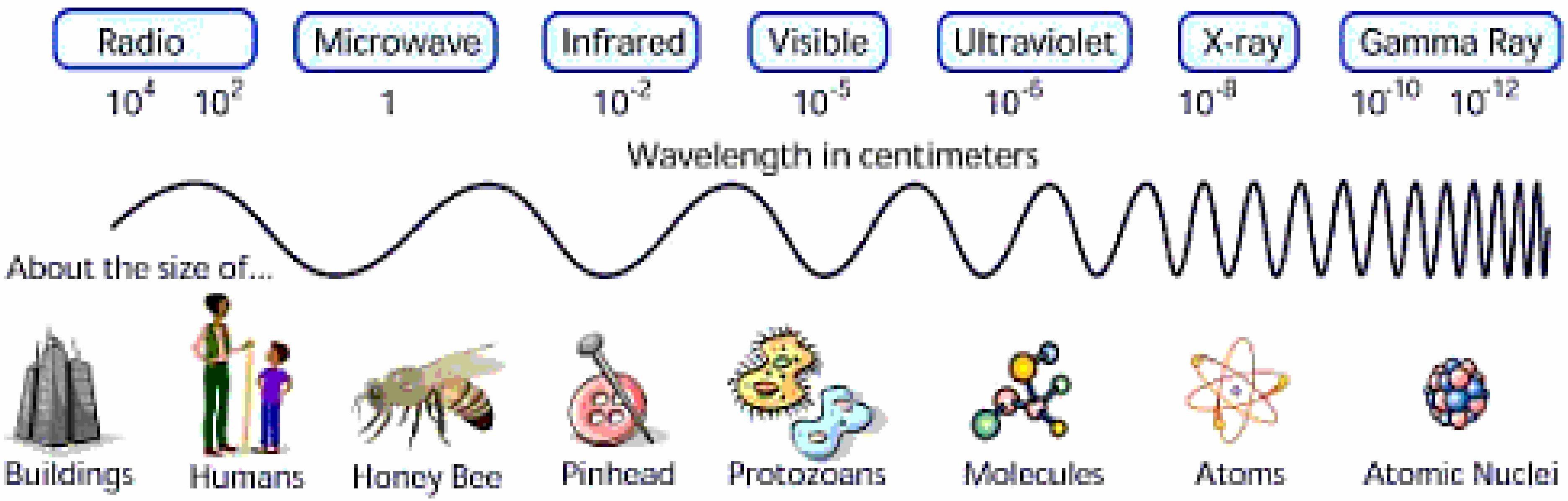
- ♦ **Basic theory**
  - ✓ Electromagnetic field
  - ✓ Maxwell equations
- ♦ **Optics in metal**


LECTRO MAGNETIC SPECTRUM
Wavelength
(metres)
Radio
Microwave
Infrared
Visible
Ultraviolet
X-Ray
Gamma Ray
10^{3}
10^{-2}
10^{-5}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Frequency
(Hz)
10^{4}
10^{8}
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}




etic Spectrum
10^{-13} cm
10^{-9} cm
10^{-6} cm
10^{-4} cm
1^{-2} cm
1 cm
1 km
Gamma ray
X-ray
Ultraviolet
Visible
Infrared
Microwave
Radio

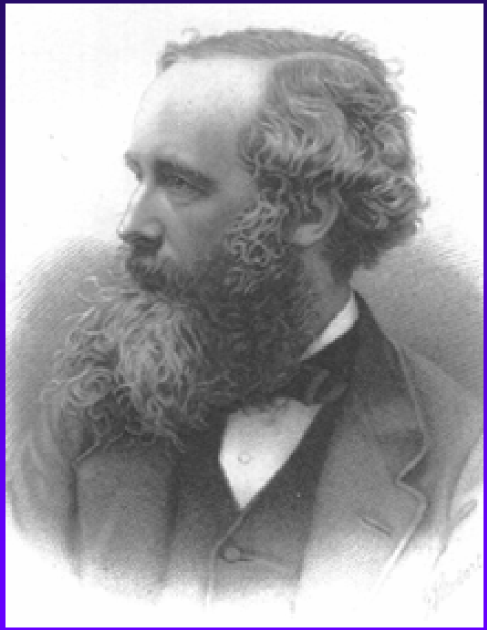
Electromagnetic waves can be described by their wavelengths, energy, and frequency. All three of these things describe a different property of light, yet they are related to each other mathematically. This means that it is correct to talk about the energy of an X-ray or the wavelength of a microwave or the frequency of a radio wave.



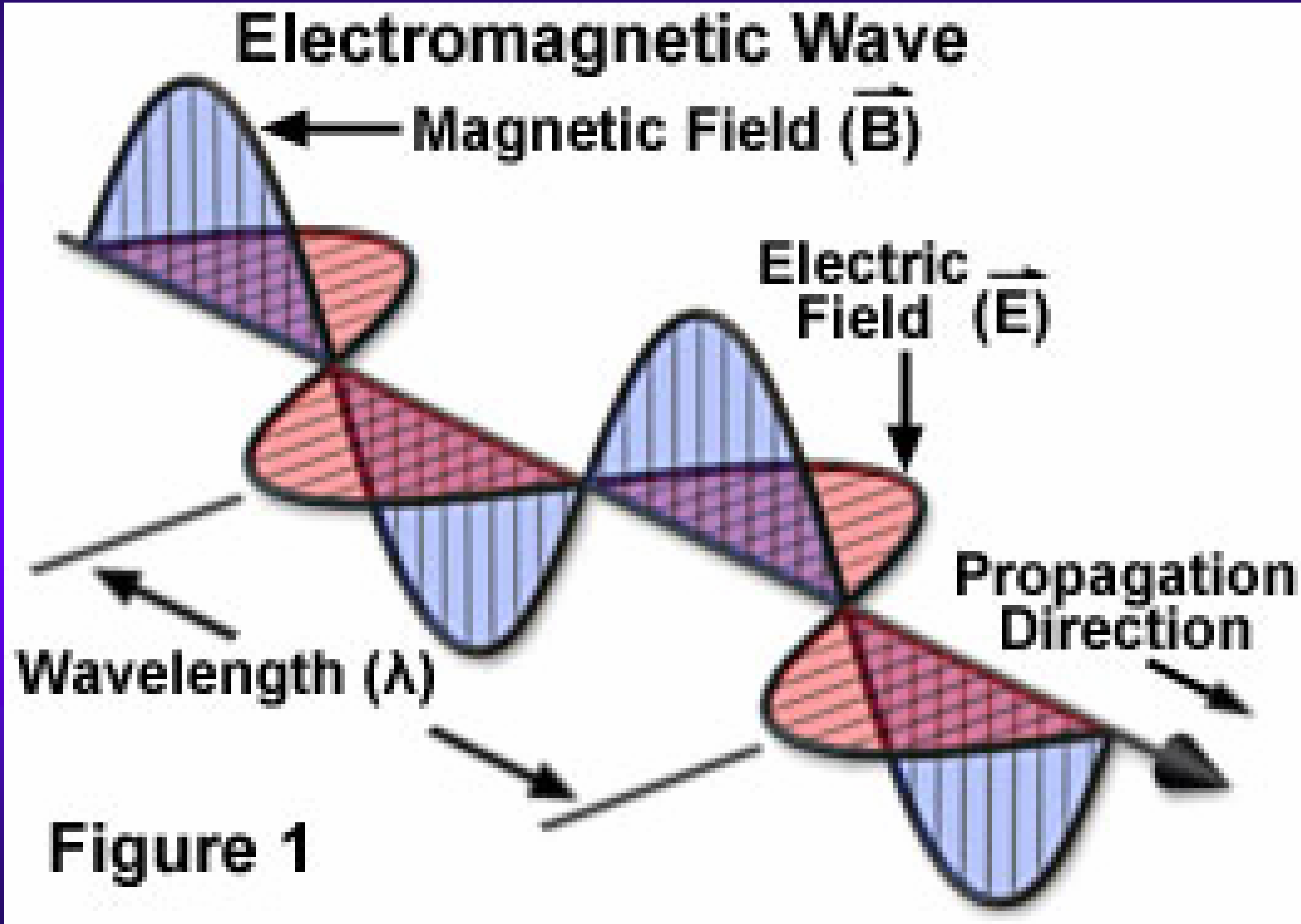
Maxwell's Equations

- Maxwell's equations represent one of the **most elegant and concise ways to state the fundamentals of electricity and magnetism**. From them one can develop most of the working relationships in the field. Because of their concise statement, they embody a high level of mathematical sophistication and are therefore not generally introduced in an introductory treatment of the subject, except perhaps as summary relationships.

Maxwell's Equations



These basic equations of electricity and magnetism can be used as a starting point for advanced courses, but are usually first encountered as unifying equations after the study of electrical and magnetic phenomena.


Electromagnetic Wave
Magnetic Field (B)
Electric Field (E)
Propagation Direction
Wavelength (λ)
Figure 1

SYMBOL

E = Electric field	ρ = charge density	i = electric current
B = Magnetic field	ε_0 = permittivity	J = current density
D = Electric displacement	μ_0 = permeability	c = speed of light
H = Magnetic field strength	M=Magnetization	P = Polarization

http://hyperphysics.phy-astr.gsu.edu/Hbase/electric/maxeq.html

Maxwell's Equations

Integral form in the absence of magnetic or polarizable media:

I. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

II. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

III. Faraday's law of induction $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

IV. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$

Maxwell's Equations

Differential form in the absence of magnetic or polarizable media:

I. Gauss' law for electricity $\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k \rho$

II. Gauss' law for magnetism $\nabla \cdot B = 0$

III. Faraday's law of induction $\nabla x\, E = -\frac{\partial B}{\partial t}$

IV. Ampere's law

$$\nabla\, x\; B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$= \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$k = \frac{1}{4\pi\varepsilon_0} =$ *Coulomb's constant* $\quad c^2 = \frac{1}{\mu_0 \varepsilon_0}$

Maxwell's Equations

Differential form with magnetic and/or polarizable media:

I. Gauss' law for electricity $\nabla \cdot D = \rho$

$$D = \varepsilon_0 E + P$$

General case

$$D = \varepsilon_0 E$$ *Free space*

$$D = \varepsilon E$$ *Isotropic linear dielectric*

II. Gauss' law for magnetism $\nabla \cdot B = 0$

III. Faraday's law of induction $\nabla x\, E = -\dfrac{\partial B}{\partial t}$

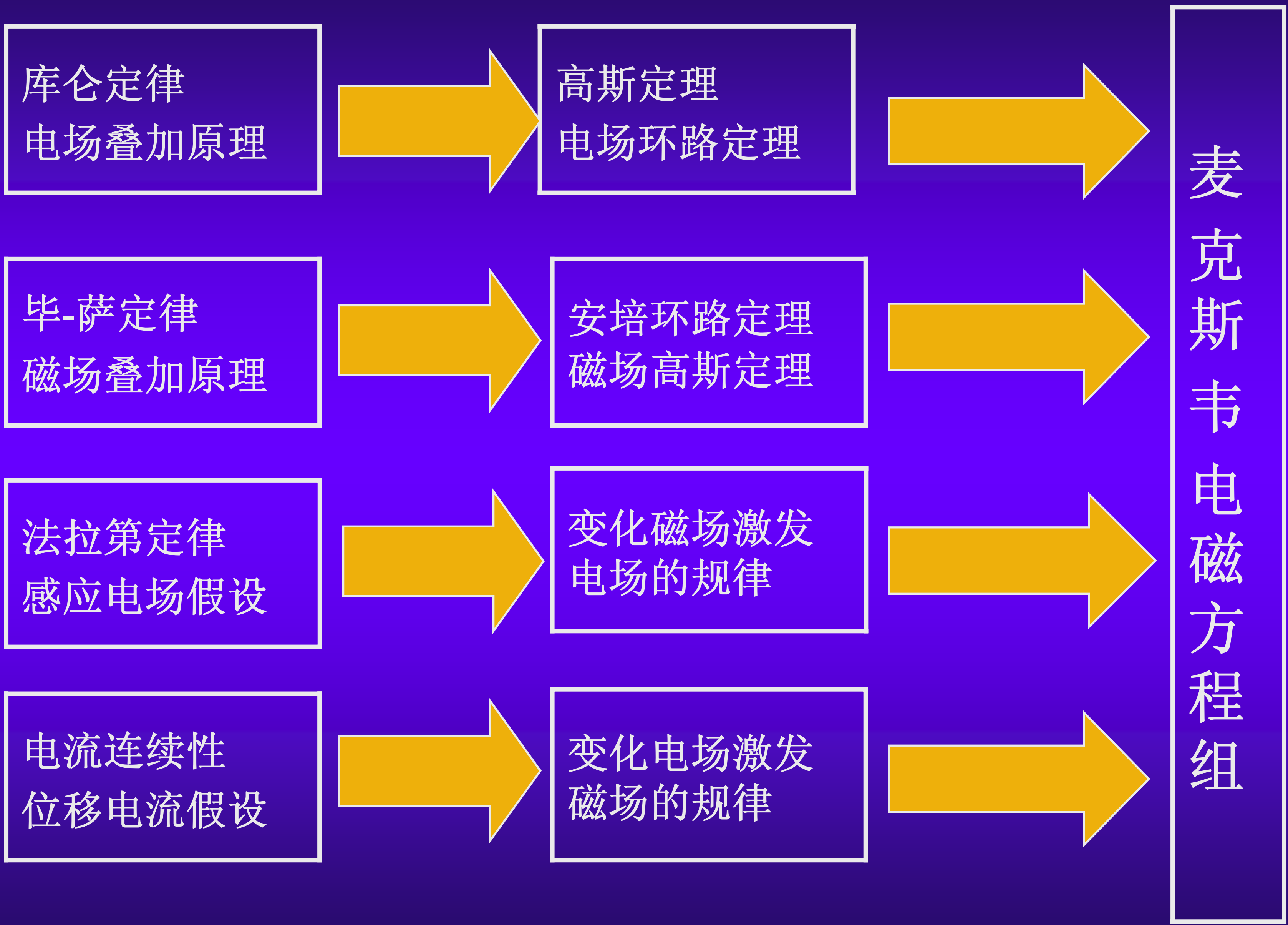
IV. Ampere's law $\nabla\, x\, H = J + \dfrac{\partial D}{\partial t}$

$$B = \mu_0 (H + M)$$

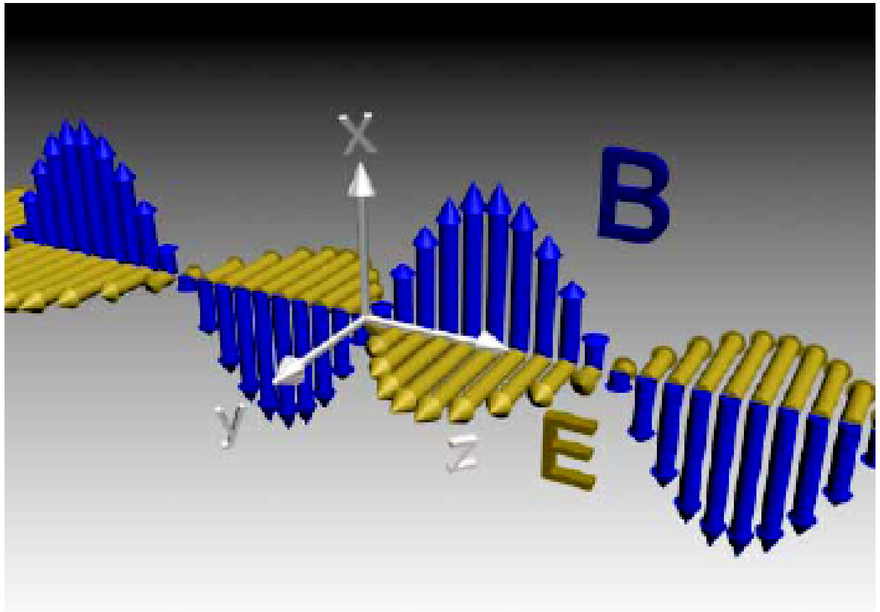
General case

$$B = \mu_0 H$$ *Free space*

$$B = \mu H$$ *Isotropic linear magnetic medium*


库仑定律
电场叠加原理
高斯定理
电场环路定理
毕-萨定律
磁场叠加原理
安培环路定理
磁场高斯定理
法拉第定律
感应电场假设
变化磁场激发
电场的规律
电流连续性
位移电流假设
变化电场激发
磁场的规律
麦克斯韦电磁方程组

Electromagnetic Wave in vacuum



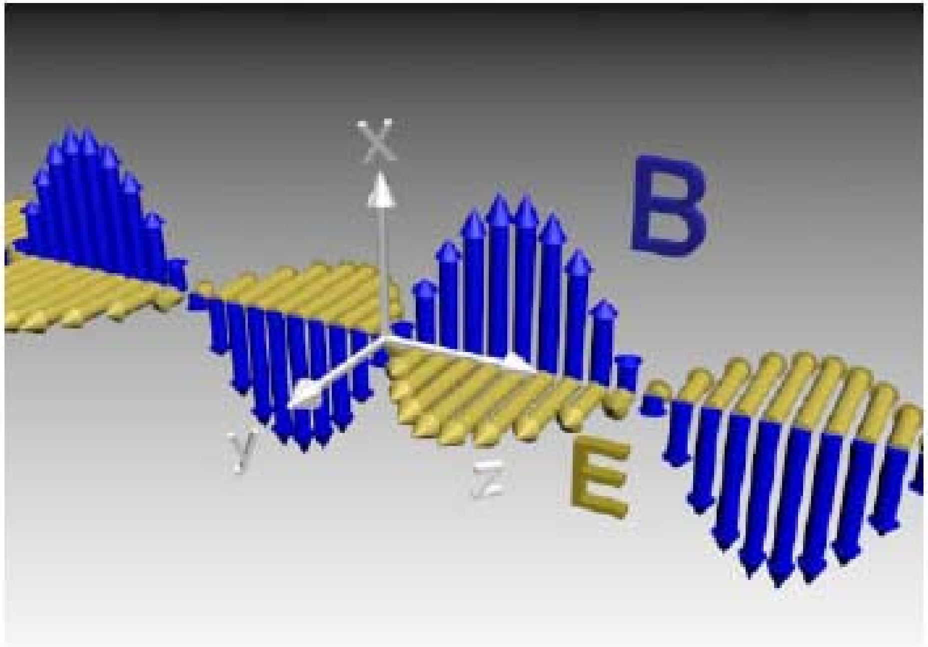
Speed of light is related to electric permittivity and magnetic permeability

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} = 3 \text{ x } 10^8 m/s$$

electric permittivity

magnetic permeability

Electromagnetic Wave in a medium



ε = permittivity
μ = permeability
o - in a vacuum
r - relative

$$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \qquad v = \frac{1}{\sqrt{\varepsilon \mu}}$$

$$n = \frac{c}{v} = \frac{\sqrt{\varepsilon \mu}}{\sqrt{\varepsilon_o \mu_o}} = \sqrt{\varepsilon_r \mu_r}$$

$$\boxed{n \cong \sqrt{\varepsilon_r}}$$

n is a material parameter for chemists !

Complex index of refraction

$$n = \sqrt{\mu_r \varepsilon_r} \approx \sqrt{\varepsilon_r}$$

$$\varepsilon_r = \left(\varepsilon'_r + \frac{i\sigma}{\omega \varepsilon_0} \right) = \varepsilon' + i\varepsilon''$$

Note the removal of the subscript

If ε_r has imaginary parts, the refractive index is:

complex: $n = n' - in''$

and ω-dependent: $n(\omega)$

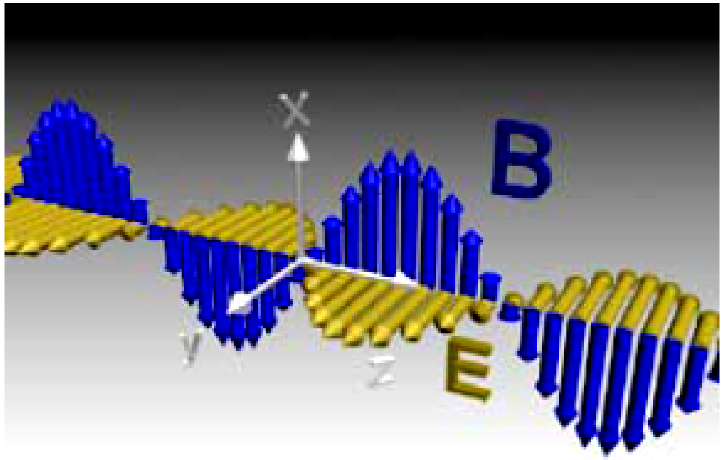
$$\boxed{n \cong \sqrt{\varepsilon_r}} \quad \Longleftrightarrow \quad \begin{aligned} n(\omega) &= n' + in'' \\ n'^2 - n''^2 &= \varepsilon' \\ 2n'n'' &= \varepsilon'' \end{aligned}$$

Electromagnetic Waves

In vacuo:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\boxed{E(x) = E_0 e^{-i\omega t} e^{ikx}}$$



In a medium:

$$k = \frac{n\omega}{c} = \frac{2n\pi}{\lambda}$$

$n = n' + in''$ ⟵ refractive index is complex

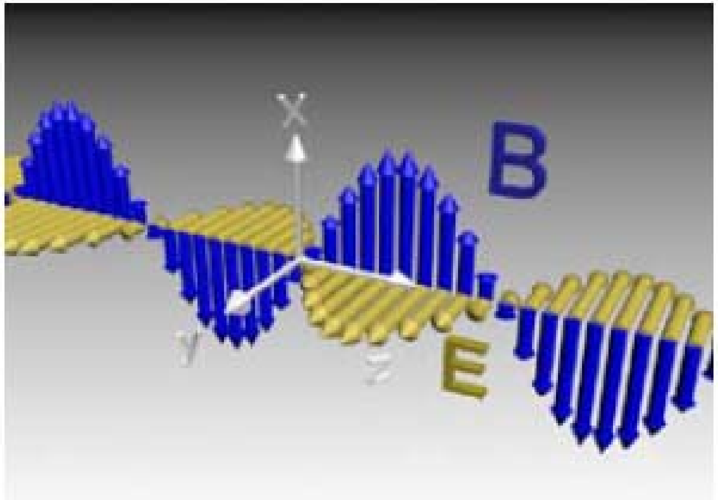
$$\boxed{n \cong \sqrt{\varepsilon_r}}$$

Electromagnetic Waves

In vacuo:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\boxed{E(x) = E_0 e^{-i\omega t} e^{ikx}}$$



In a medium:

$$k = \frac{n\omega}{c} = \frac{2n\pi}{\lambda}$$

$$n = n' + in''$$

$$\boxed{n \cong \sqrt{\varepsilon_r}}$$

$$E(x) = E_0 e^{-i\omega t} e^{\frac{i\omega(n'+in'')x}{c}}$$

$$E(x) = E_0 e^{\left(\frac{i\omega n'x}{c} - i\omega t\right)} e^{\frac{-\omega n''x}{c}}$$

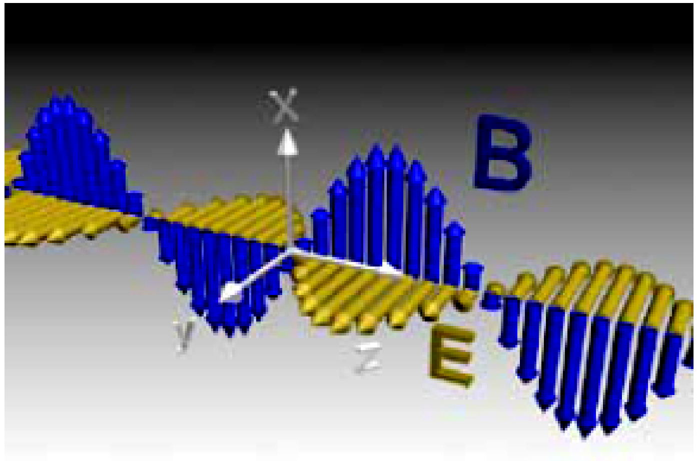
$$\boxed{E(x) = E_0 e^{\left(\frac{i2\pi n'x}{\lambda} - i\omega t\right)} e^{\frac{-2\pi n''x}{\lambda}}}$$

Electromagnetic Waves

In vacuo:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\boxed{E(x) = E_0 e^{-i\omega t} e^{ikx}}$$



In a medium:

$$k = \frac{n\omega}{c} = \frac{2n\pi}{\lambda}$$

$$n = n' + in''$$

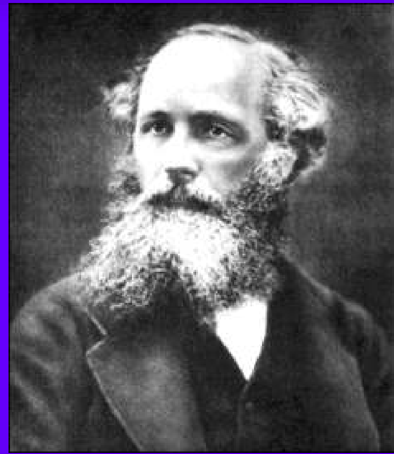
$$\boxed{n \cong \sqrt{\varepsilon_r}}$$

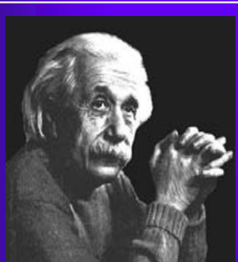
Decay of wave amplitude

Phase velocity

$$\boxed{E(x) = E_0 e^{\left(\frac{i2\pi n' x}{\lambda} - i\omega t\right)} e^{\frac{-2\pi n'' x}{\lambda}}}$$

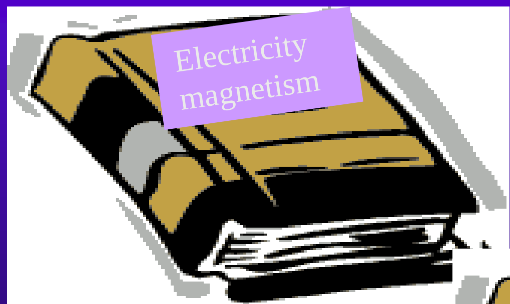
Games Clerk Maxwell (1831-1879): Interplay of electric and magnetic field could result in electromagnetic waves (1860)

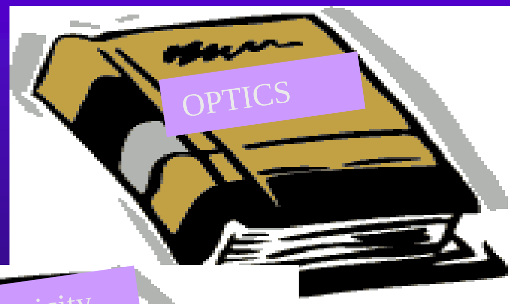




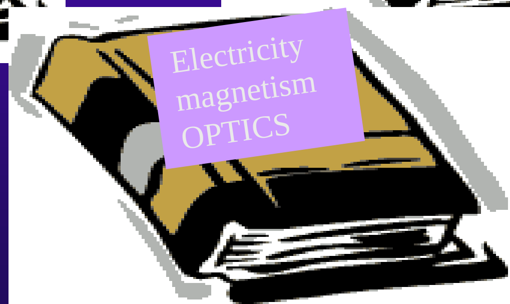
"Maxwell´s accomplishments are the most profound and the most fruitful that physics has experienced since the time of Newton " (A. Einstein):

Before Maxwell:





After Maxwell:



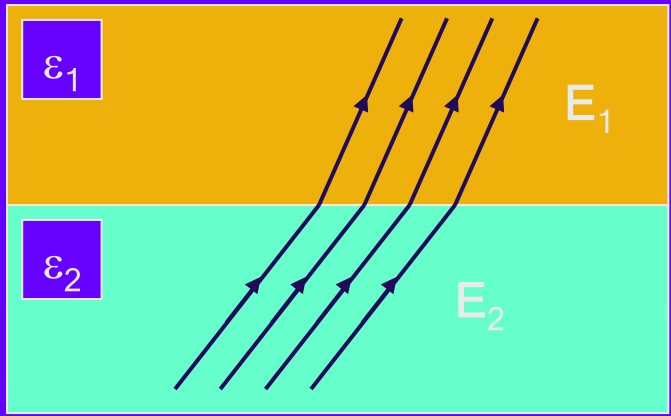
Maxwell’s equations

Optics wave
- Charge neutrality, $\rho = 0$
- No direct current, $j = 0$
- Nonmagnetic materials, $\mu_r = 1$ ($\mu = \mu_0$)

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \cancel{\frac{\rho}{\varepsilon}} \\
\nabla \cdot \mathbf{H} &= 0 \\
\nabla \times \mathbf{E} &= -\cancel{\mu} \frac{\partial \mathbf{H}}{\partial t} \\
\nabla \times \mathbf{H} &= \cancel{\mathbf{j}} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\nabla \cdot \mathbf{E} &= 0 \\
\nabla \cdot \mathbf{H} &= 0 \\
\nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \\
\nabla \times \mathbf{H} &= \varepsilon \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

Boundary conditions

In inhomogeneous media consisting of several dielectrics, the field lines of E, H will experience discontinuity or bending at the boundary



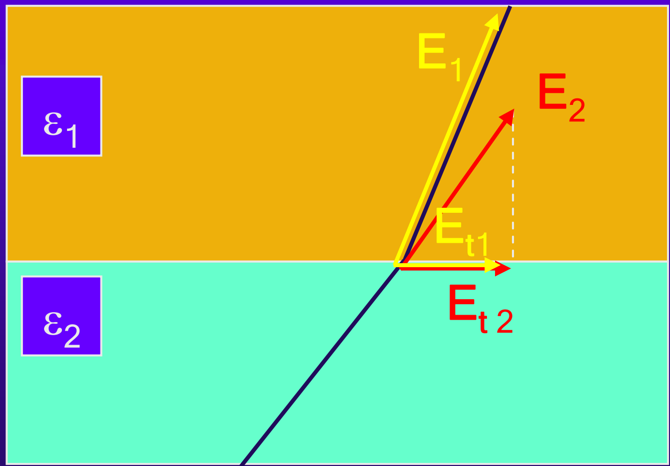
The boundary conditions for **E**, **H** can be derived from Maxwell equations

normal components:

$$\begin{aligned} D_{n_1} &= D_{n_2} \\ B_{n_1} &= B_{n_2} \end{aligned}$$

tangential components:

$$\begin{aligned} E_{t_1} &= E_{t_2} \\ H_{t_1} &= H_{t_2} \end{aligned}$$



Electromagnetic waves

Maxwell's wave equations: (in vacuum)

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E}{\partial x^2}$$

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 B}{\partial x^2}$$

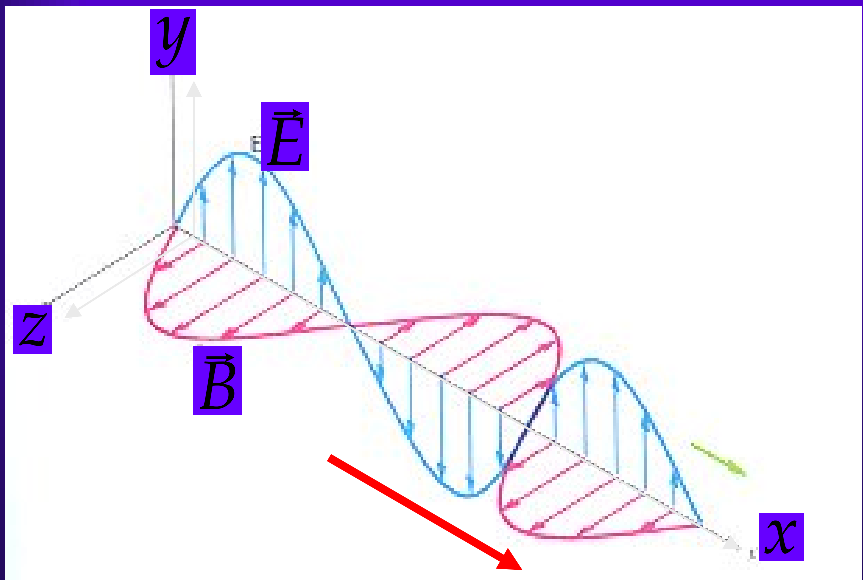
$E =$ electric field

$B =$ magnetic field

$\varepsilon_0 =$ permittivity (vacuum)

$\mu_0 =$ permeability (vacuum)

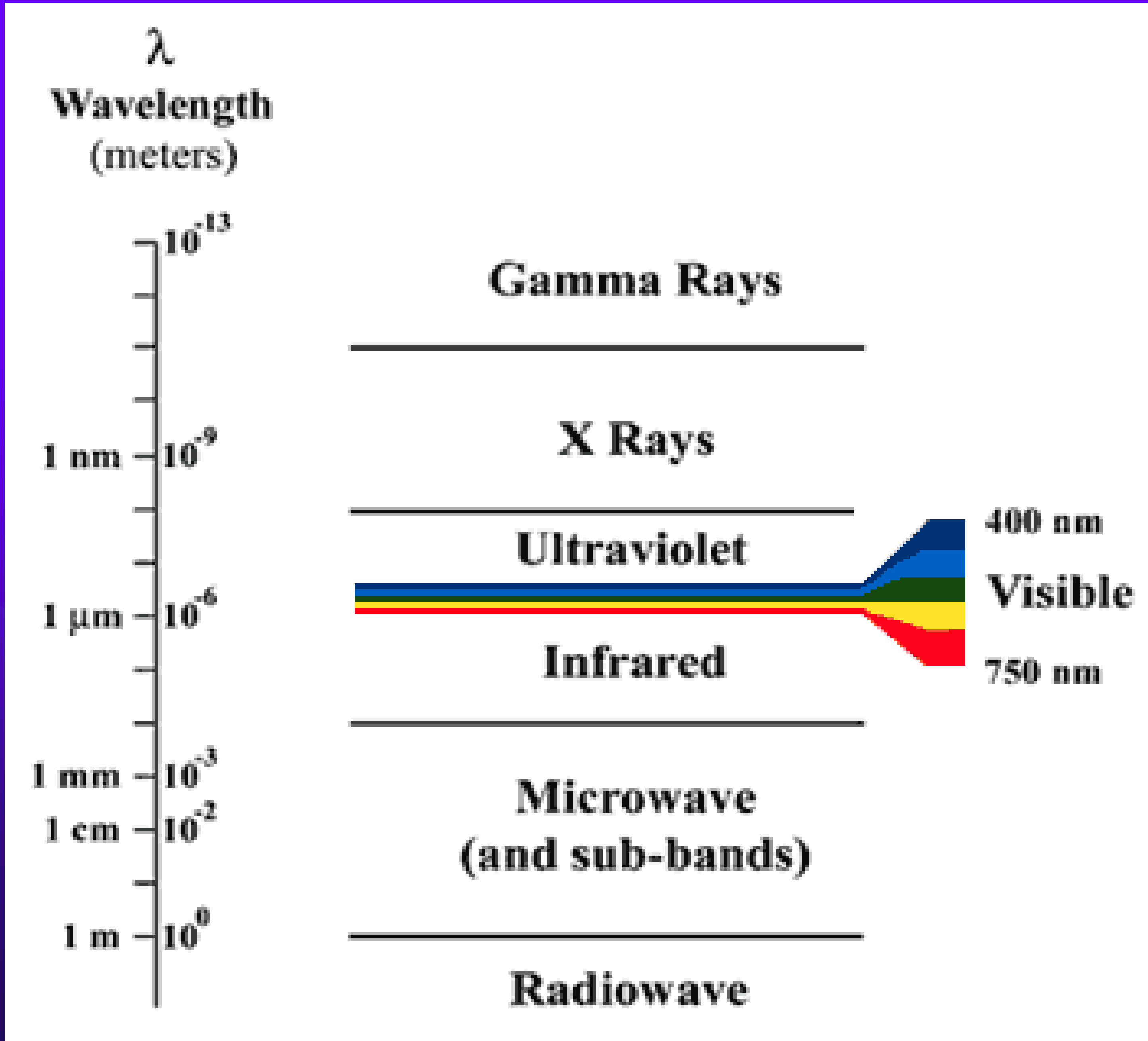
speed of light $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s



$$E(x,t) = E_0 \sin(\omega t - kx)$$

$$B(x,t) = B_0 \sin(\omega t - kx)$$

The electromagnetic spectrum



Electromagnetic waves in matter

vacuum: $$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\boxed{\mu_0 \varepsilon_0}} \frac{\partial^2 E}{\partial x^2} \qquad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\boxed{\mu_0 \varepsilon_0}} \frac{\partial^2 B}{\partial x^2}$$

matter: $$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\boxed{\mu \varepsilon}} \frac{\partial^2 E}{\partial x^2} \qquad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\boxed{\mu \varepsilon}} \frac{\partial^2 B}{\partial x^2}$$

permittivity : $\varepsilon = \varepsilon_r \varepsilon_0$ (ε_r = dielectric constant)

permeability : $\mu = \varepsilon_r \varepsilon_0$ (μ_r = relative permeability; $\mu_r \approx 1$)

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \, \varepsilon_r \varepsilon_0}} = \underbrace{\boxed{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}}_{= c} \boxed{\frac{1}{\sqrt{\mu_r \varepsilon_r}}} = n \text{ refraction index}$$

$$\boxed{v = \frac{c}{n}}$$

The 3D Vector Wave Equation for Electric Field

$$\vec{\nabla}^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Derived from Maxwell's Equations

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

This is just 3 independent wave equations, one for each x-, y-, and z-components of E.

which has the vector field solution:

$$\vec{\mathrm{E}}(\vec{\mathrm{r}}, t) = E_0 e^{i(\vec{\mathrm{k}} \cdot \vec{\mathrm{r}} - \omega t)} \Longrightarrow \vec{\mathrm{E}}(\vec{\mathrm{r}}, t) = \mathrm{E}(\vec{\mathrm{r}}) e^{-i\omega t}$$

Vector Helmholtz Equation

- Helmholtz Equation in free space derived from wave equation

$$\boxed{\nabla^2 \vec{\mathrm{E}}(\vec{\mathrm{r}}) + k^2 \varepsilon_0 \vec{\mathrm{E}}(\vec{\mathrm{r}}) = 0}$$

where $k = \dfrac{2\pi}{\lambda}$

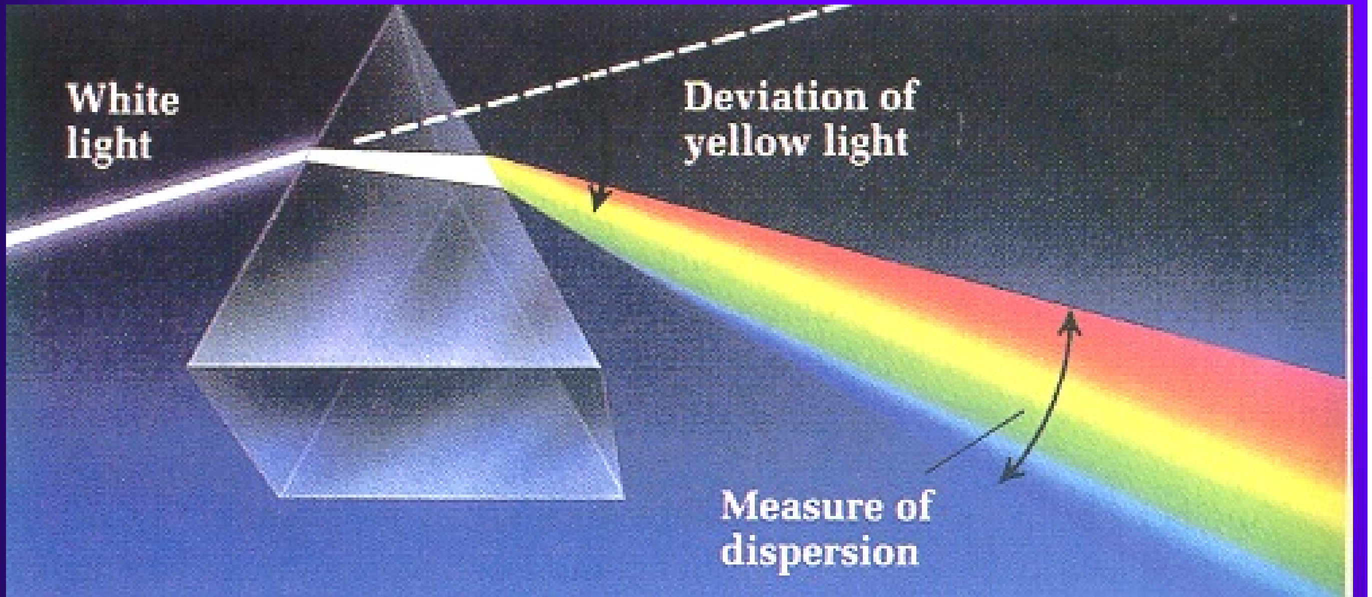
and $\varepsilon_0 =$ *free space permittivity*

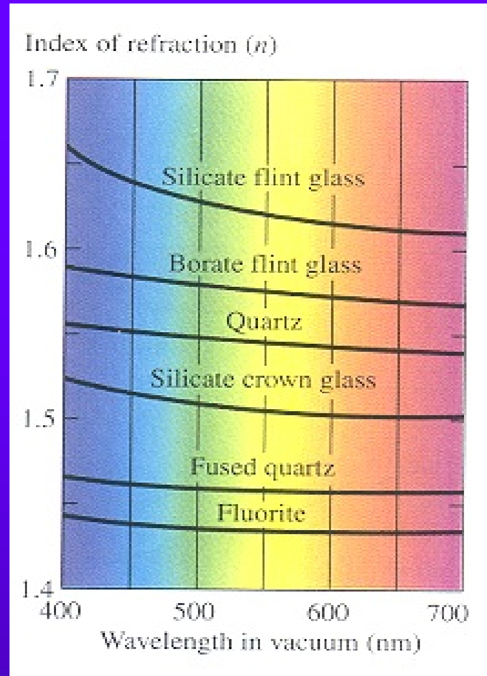
$$\vec{\mathrm{E}}(\vec{\mathrm{r}}) = \left(\mathrm{E}_x, \mathrm{E}_y, \mathrm{E}_z\right)$$

$$\vec{\mathrm{E}}(\vec{\mathrm{r}}) = (\underbrace{\mathrm{Re}[E_x] + i\,\mathrm{Im}[E_x]}_{x\text{-component}}, \underbrace{\mathrm{Re}[E_y] + i\,\mathrm{Im}[E_y]}_{y\text{-component}}, \underbrace{\mathrm{Re}[E_z] + i\,\mathrm{Im}[E_z]}_{z\text{-component}})$$

- The complex electric field has six numbers that must to be specified to completely determine its value

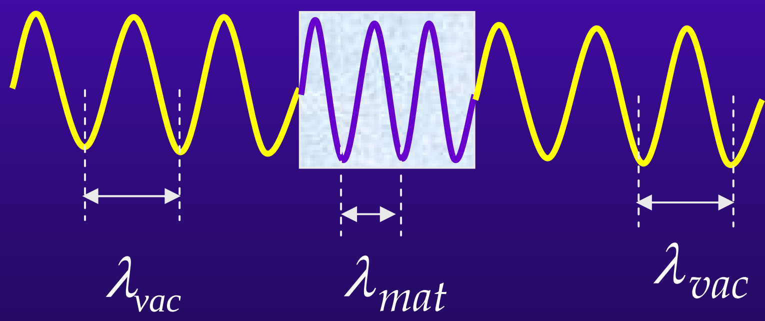
The dependence of the wave speed v and index of refraction n on the wavelength λ is called dispersion



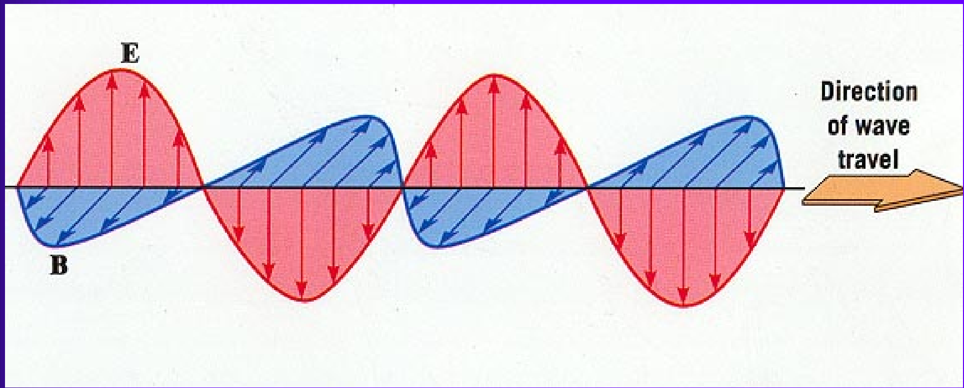


What is the wavelength of light in a medium with the refractive index *n*?

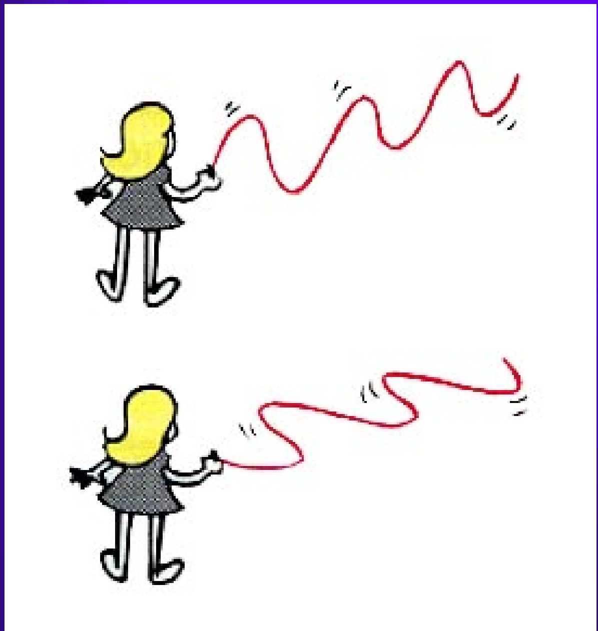
$$\lambda_{mat} = \frac{v}{c}\lambda_{vac} = \frac{\lambda_{vac}}{n}$$

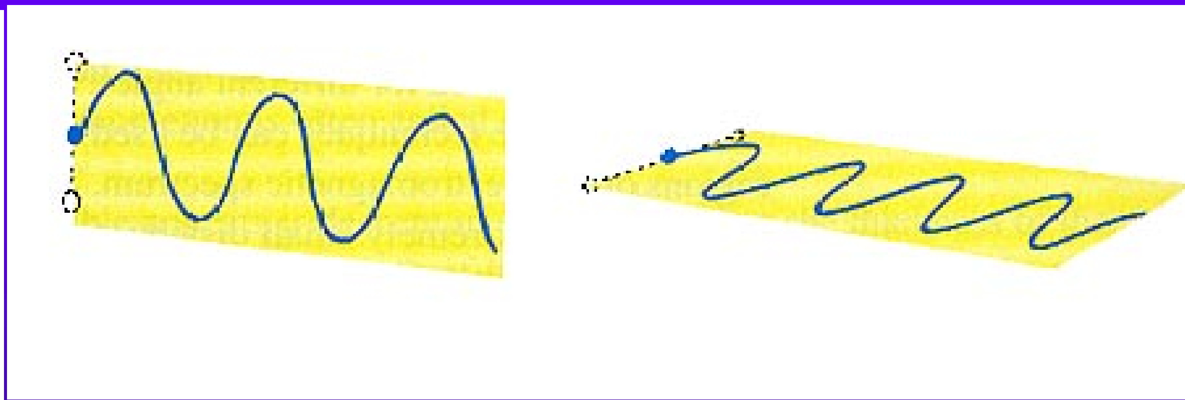


POLARIZATION



Plane Polarized Electromagnetic Waves

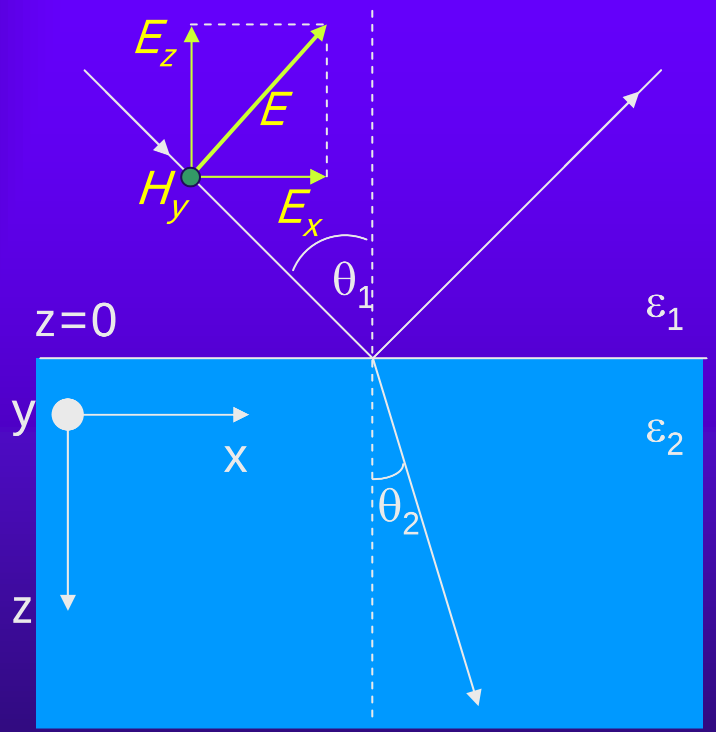




Illustrating vertical and horizontal polarized waves.

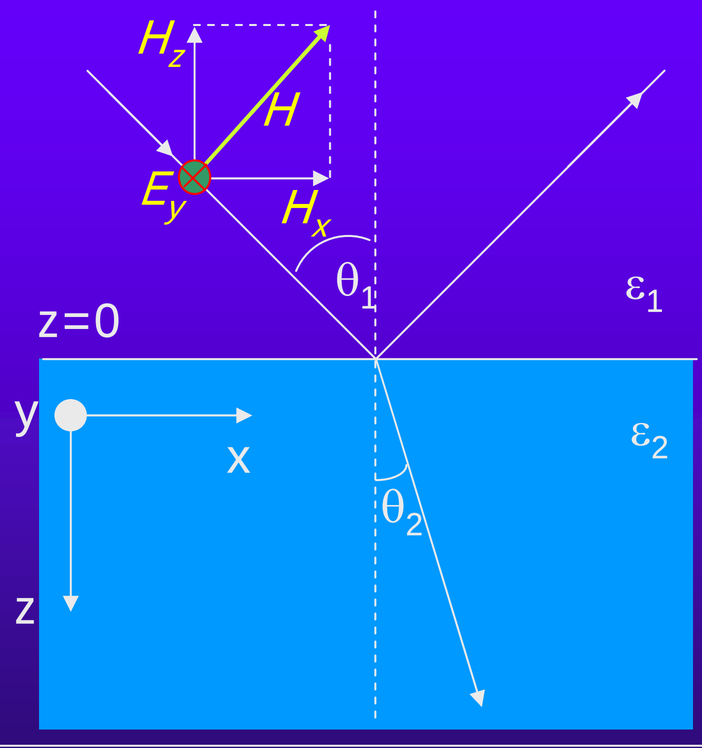
p-polarization:

E-field is parallel to the plane of incidence



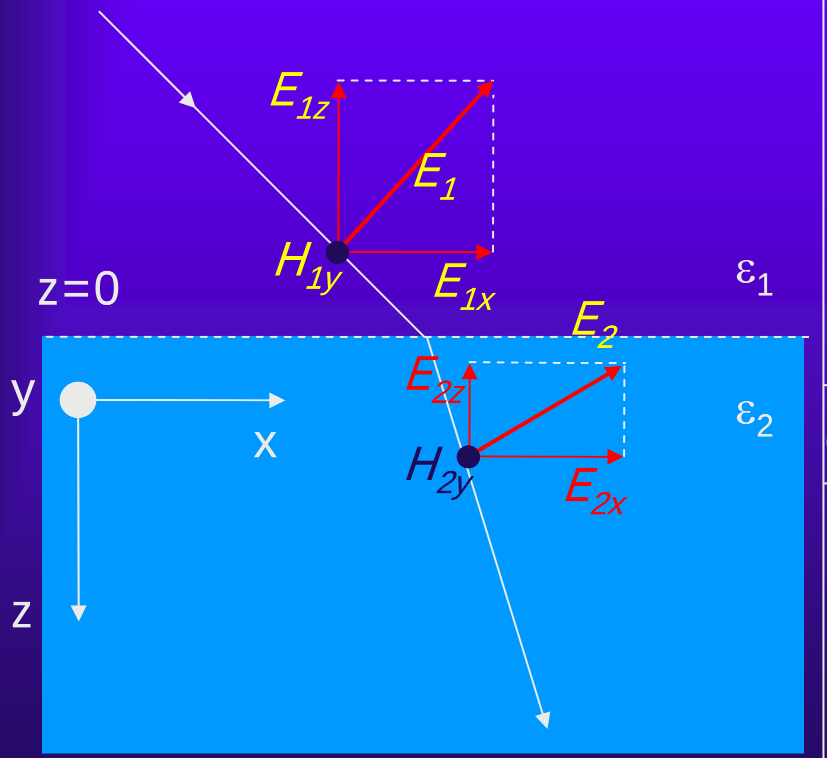
s-polarization:

E-field is perpendicular to the plane of incidence



Any linearly polarized radiation can be represented as a superposition of p- and s-polarization.

p-polarized incident radiation will create polarization charges at the interface. We will show that these charges give rise to a surface plasmon modes



Boundary condition:

(a) transverse component of **E** is conserved,

$$E_{1x} = E_{2x}$$

(b) normal component of **D** is conserved

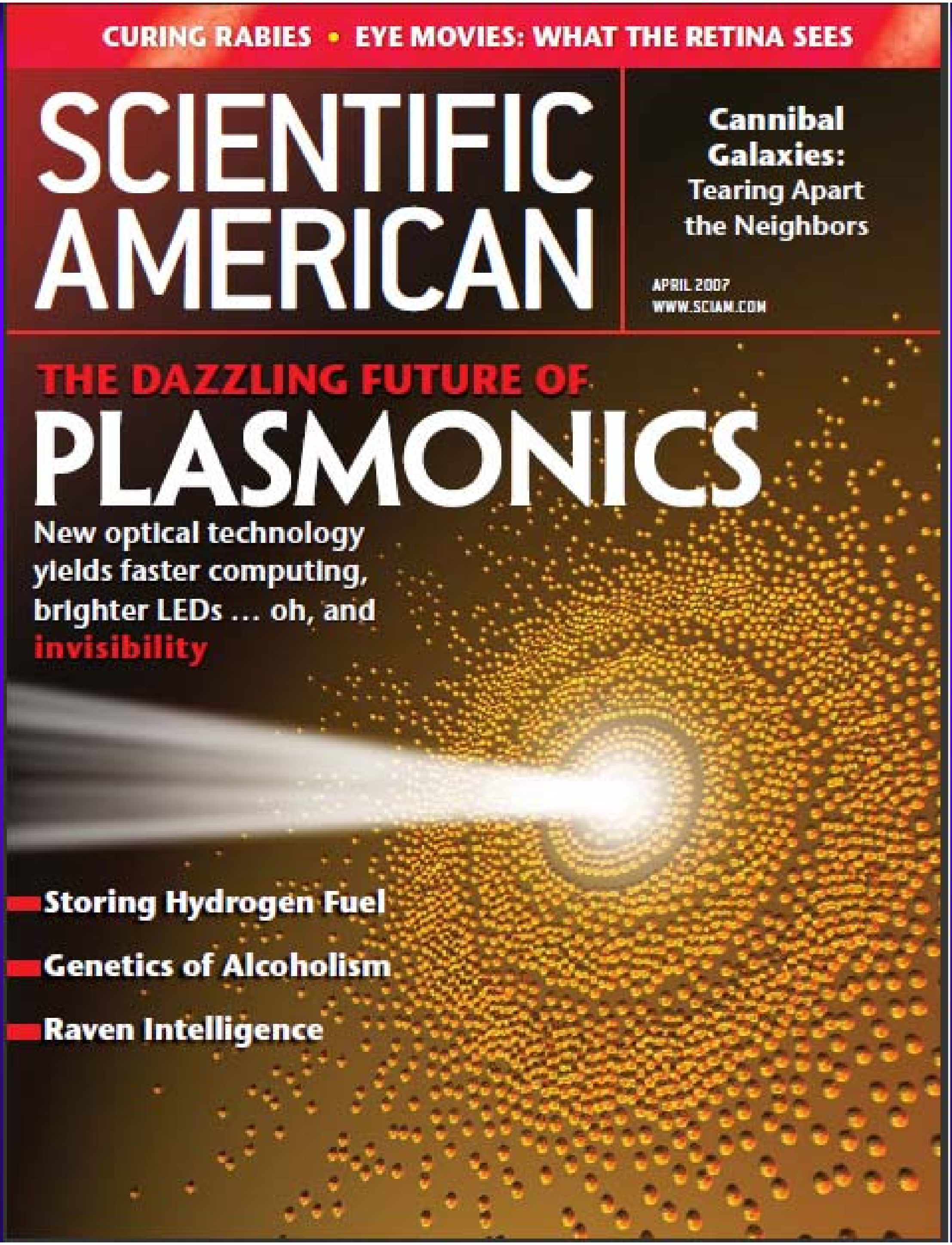
$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$$

$$D_{1z} = D_{2z}$$

$$\varepsilon_0 E_{0z} + P_{1z} = \varepsilon_0 E_{0z} + P_{2z}$$

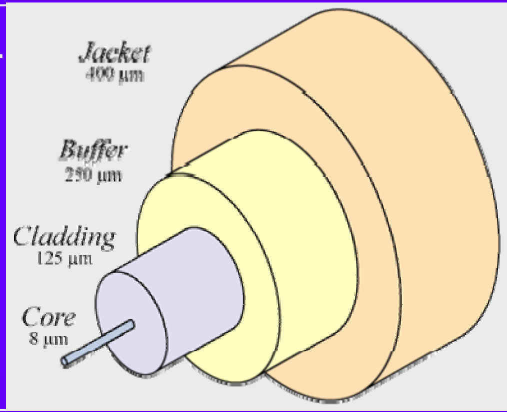
creation of the polarization charges

if one of the materials is metal, the electrons will respond to this polarization. This will give rise to surface plasmon modes

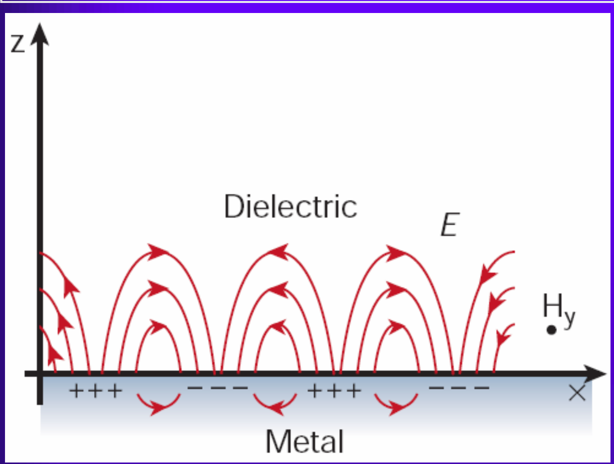

CURING RABIES • EYE MOVIES: WHAT THE RETINA SEES
SCIENTIFIC AMERICAN
Cannibal Galaxies: Tearing Apart the Neighbors
APRIL 2007
WWW.SCIAM.COM
THE DAZZLING FUTURE OF
PLASMONICS
New optical technology yields faster computing, brighter LEDs … oh, and invisibility
Storing Hydrogen Fuel
Genetics of Alcoholism
Raven Intelligence

MOTIVATION

optical fiber



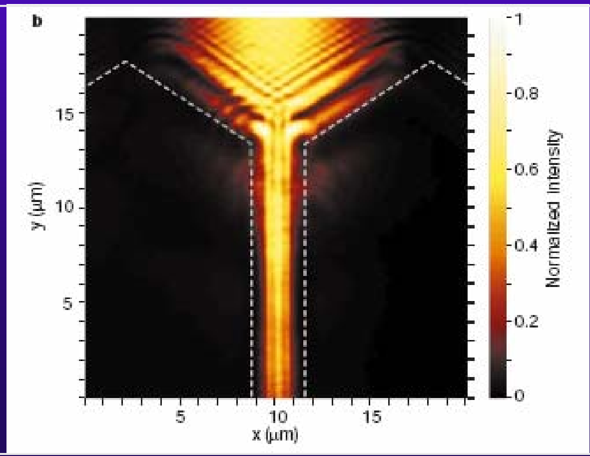
The miniaturization of conventional photonic circuits is limited by the diffraction limit, such that the minimum feature size is of the order of wavelength.

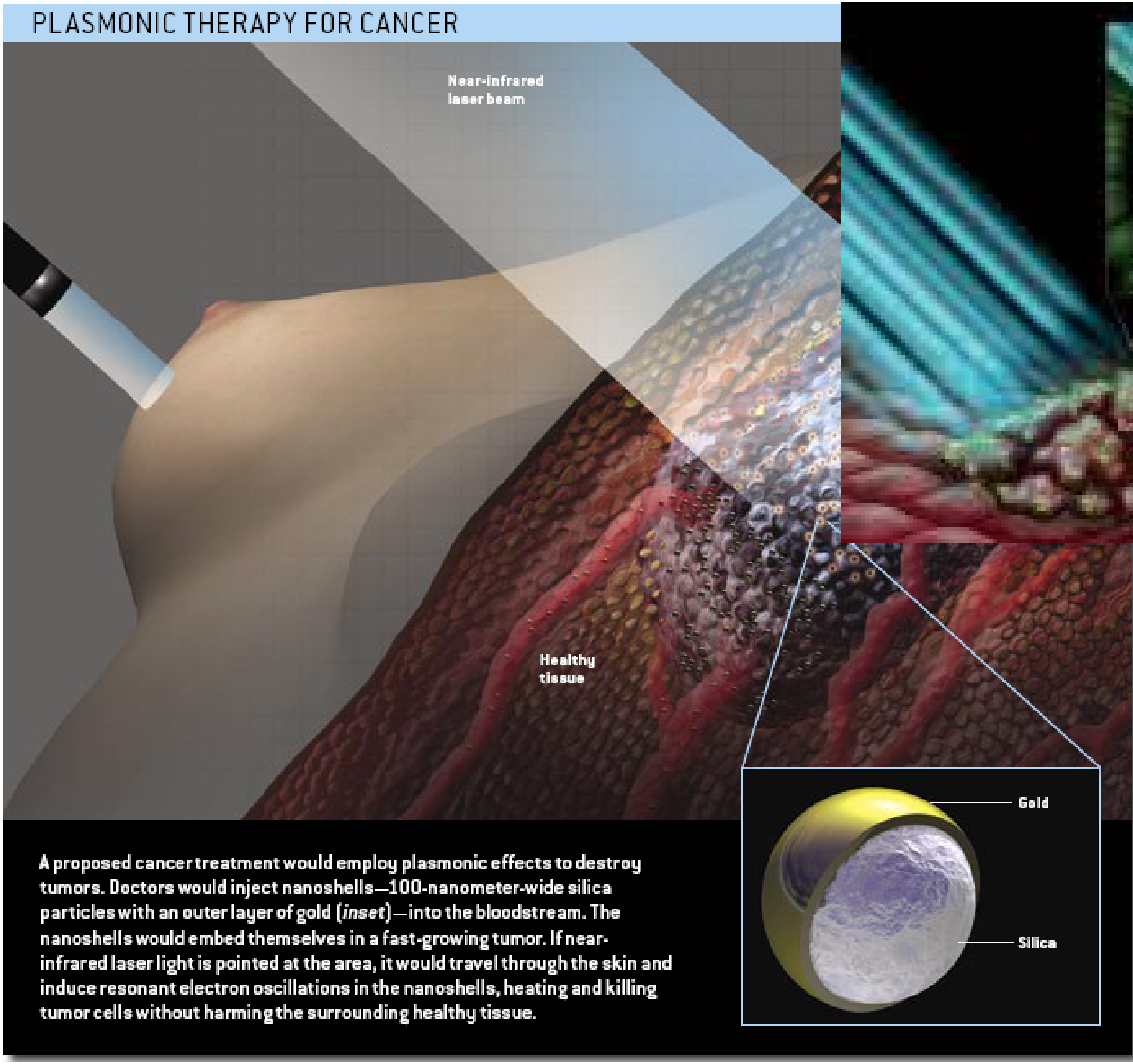


- ✓ Surface plasmons have a combined electromagnetic wave and surface charge character.
- ✓ They reside at the interface between a metal and a dielectric material.

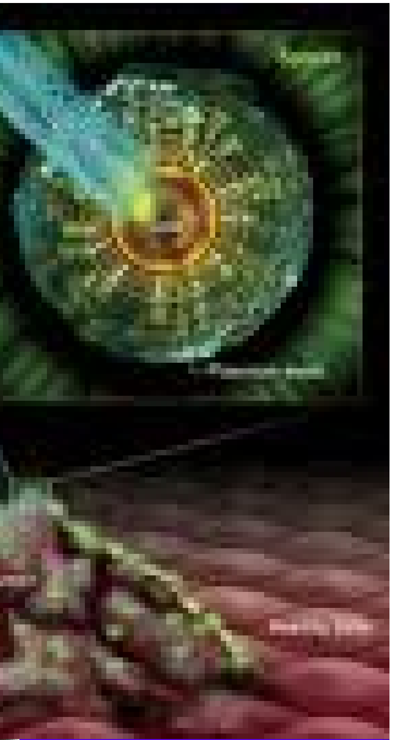
Using the surface plasmons, one can overcome the diffraction limit, which can lead to miniaturization of photonics circuits with length scales much smaller than those currently achieved

light propagation in a plasmonic waveguide



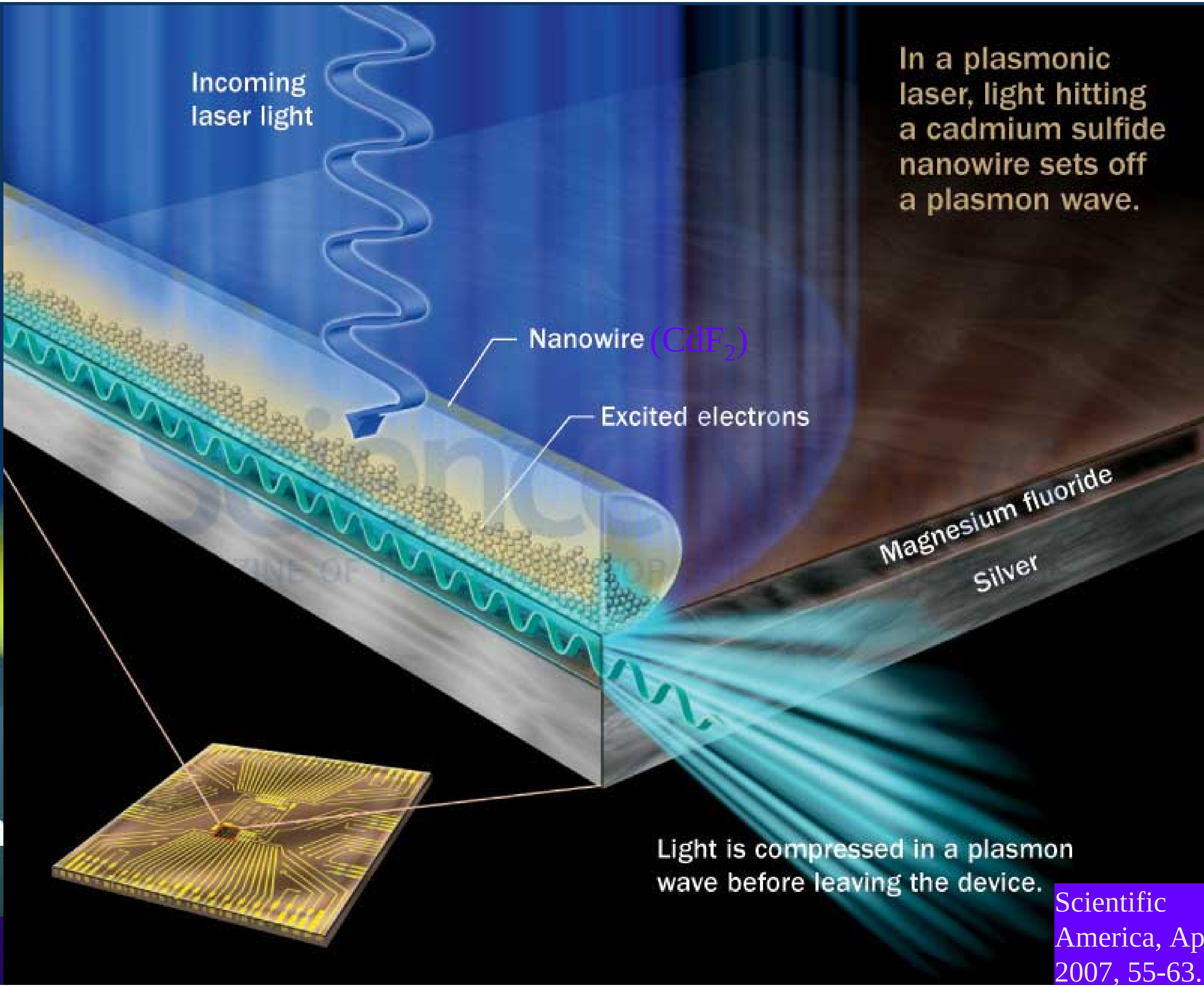


A proposed cancer treatment would employ plasmonic effects to destroy tumors. Doctors would inject nanoshells—100-nanometer-wide silica particles with an outer layer of gold (*inset*)—into the bloodstream. The nanoshells would embed themselves in a fast-growing tumor. If near-infrared laser light is pointed at the area, it would travel through the skin and induce resonant electron oscillations in the nanoshells, heating and killing tumor cells without harming the surrounding healthy tissue.



Opto-thermal therapy

Scientific America, April 2007, 55-63.



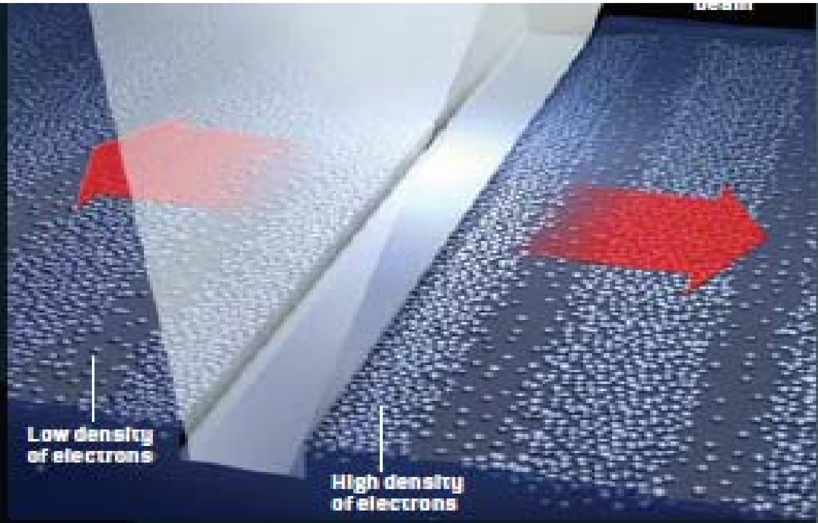
In

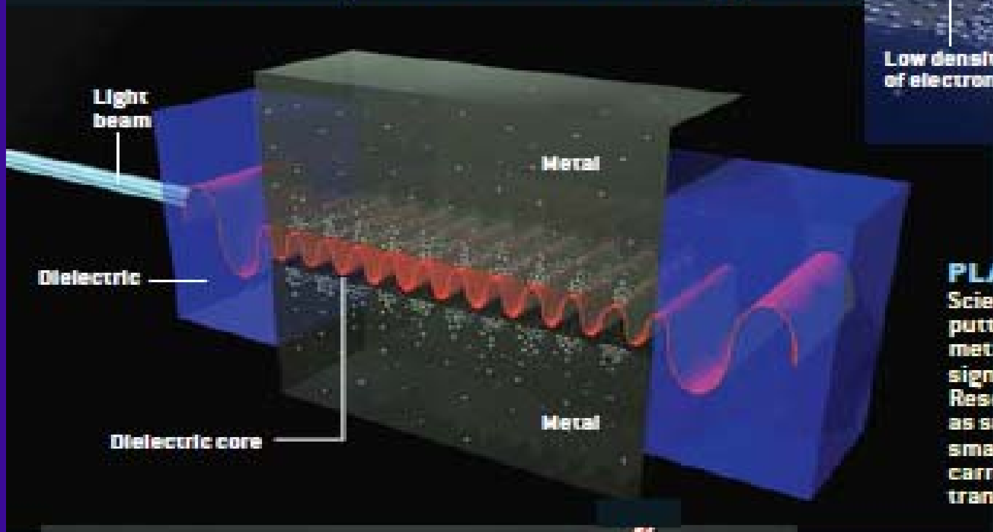
Scientific America, April 2007, 55-63.

that demonstrate the promise of the technology.

PLANAR WAVEGUIDE

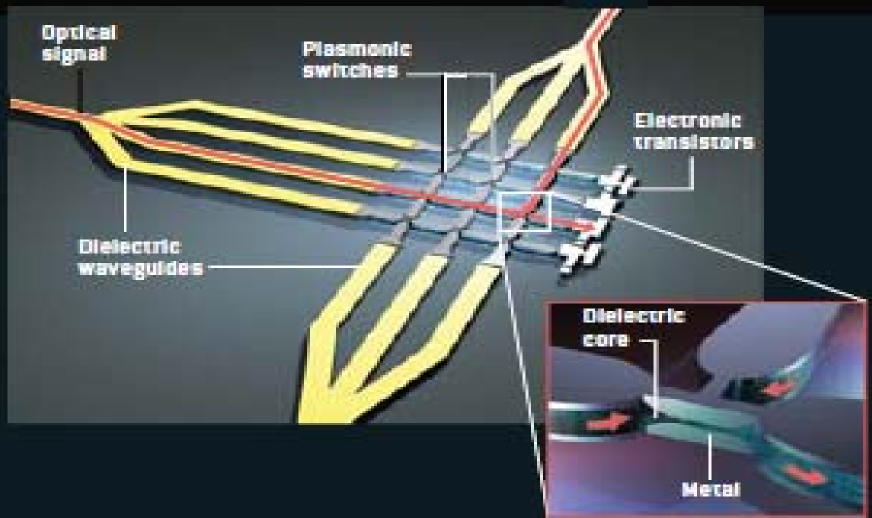
Plasmons always flow along the boundary between a metal and a dielectric (a nonconductive material such as air or glass). For example, light focused on a straight groove in a metal will generate plasmons that propagate in the thin plane at the metal's surface (the boundary between the metal and air). A plasmon could travel as far as several centimeters in this planar waveguide—far enough to convey a signal from one part of a chip to another—but the relatively large wave would interfere with other signals in the nanoscale innards of a processor.





PLASMON SLOT WAVEGUIDE

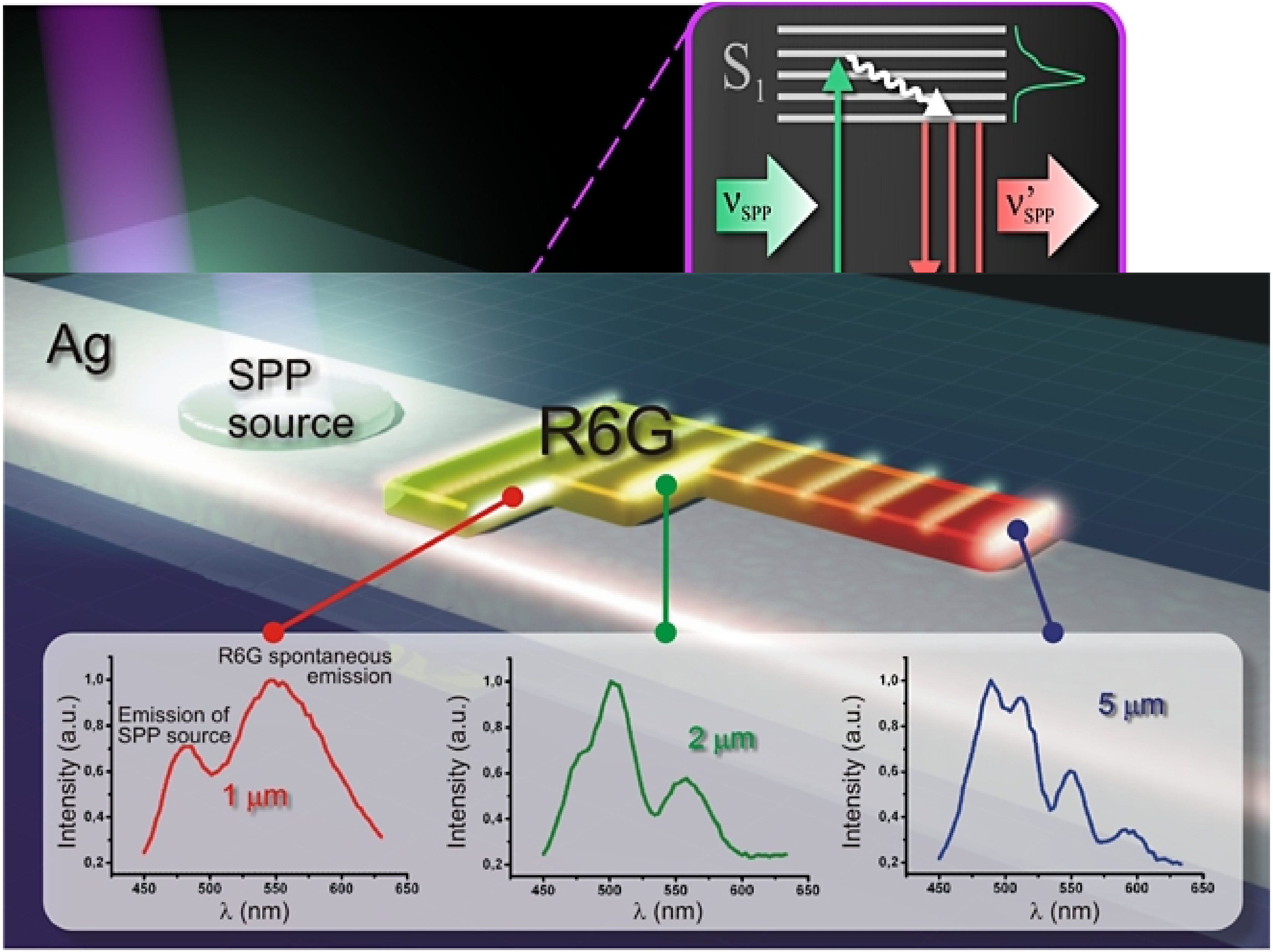
Scientists have built much smaller plasmonic circuits by putting the dielectric at the core and surrounding it with metal. The plasmon slot waveguide squeezes the optical signal, shrinking its wavelength by a factor of 10 or more. Researchers have constructed slot waveguides with widths as small as 50 nanometers—about the same size as the smallest electronic circuits. The plasmonic structure can carry much more data than an electronic wire, but it cannot transmit a signal farther than 100 microns.



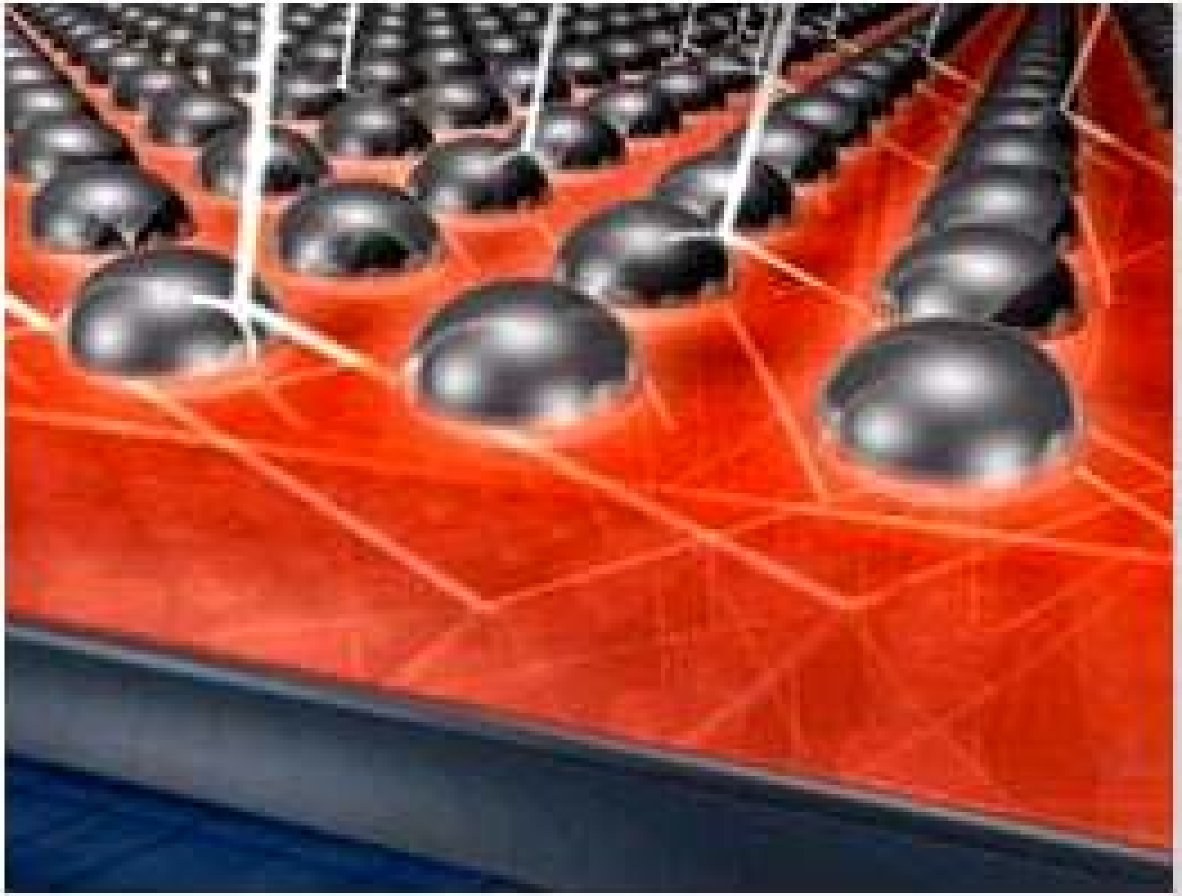
A FASTER CHIP

Slot waveguides could significantly boost the speed of computer chips by rapidly funneling large amounts of data to the circuits that perform logical operations. In the rendering at the left, relatively large dielectric waveguides deliver optical signals to an array of plasmonic switches (dubbed "plasmonsters"), which in turn distribute the signals to electronic transistors. The plasmonsters are composed of slot waveguides that measure 100 nanometers across at their broadest points and only 20 nanometers across at intersections (*inset*).

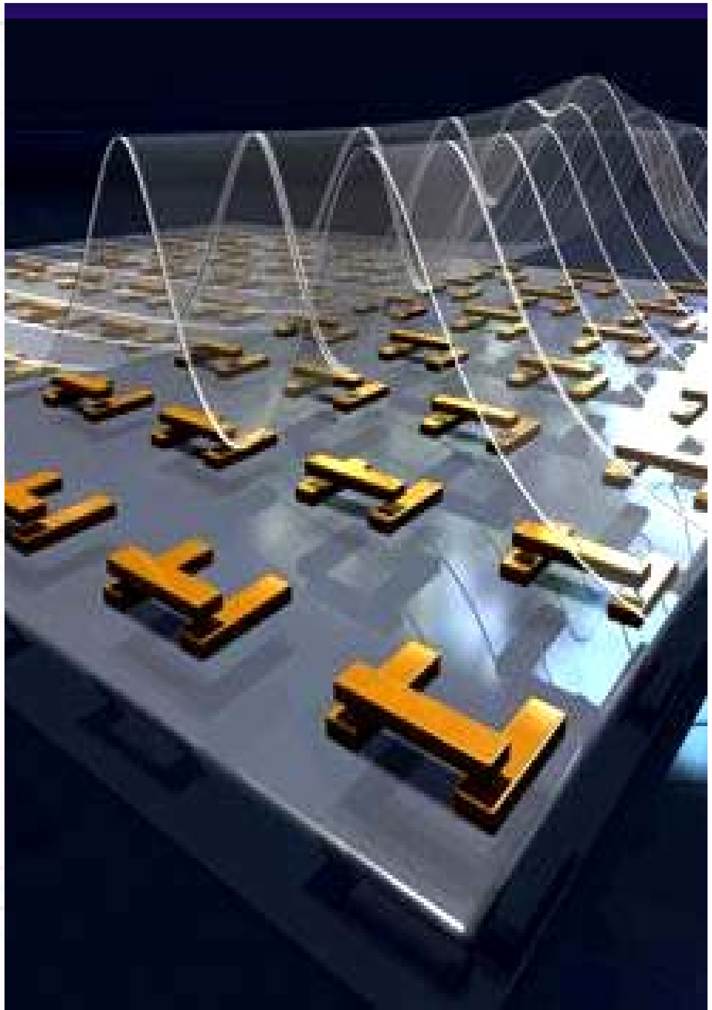
Scientific America, April 2007, 55-63.



S1
νSPP
ν'SPP
Ag
SPP source
R6G
R6G spontaneous emission
Emission of SPP source
1 µm
2 µm
5 µm
Intensity (a.u.)
λ (nm)



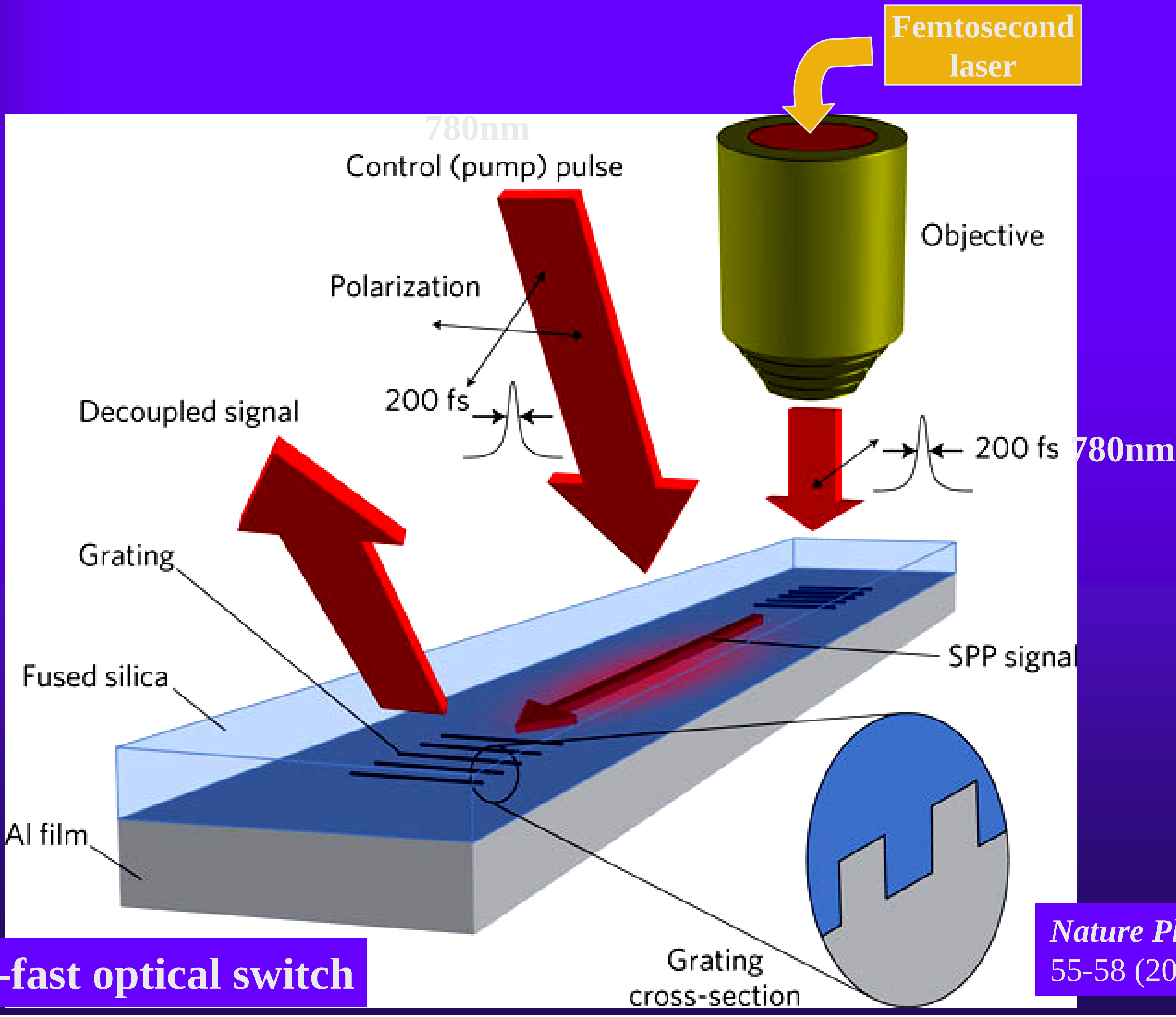
Light manipulation: surface plasmons could be generated to help direct light using nanoantennas in devices such as solar cells.



SP can be used as biomedical sensor. Resonate curves indicate absorption spectrum.

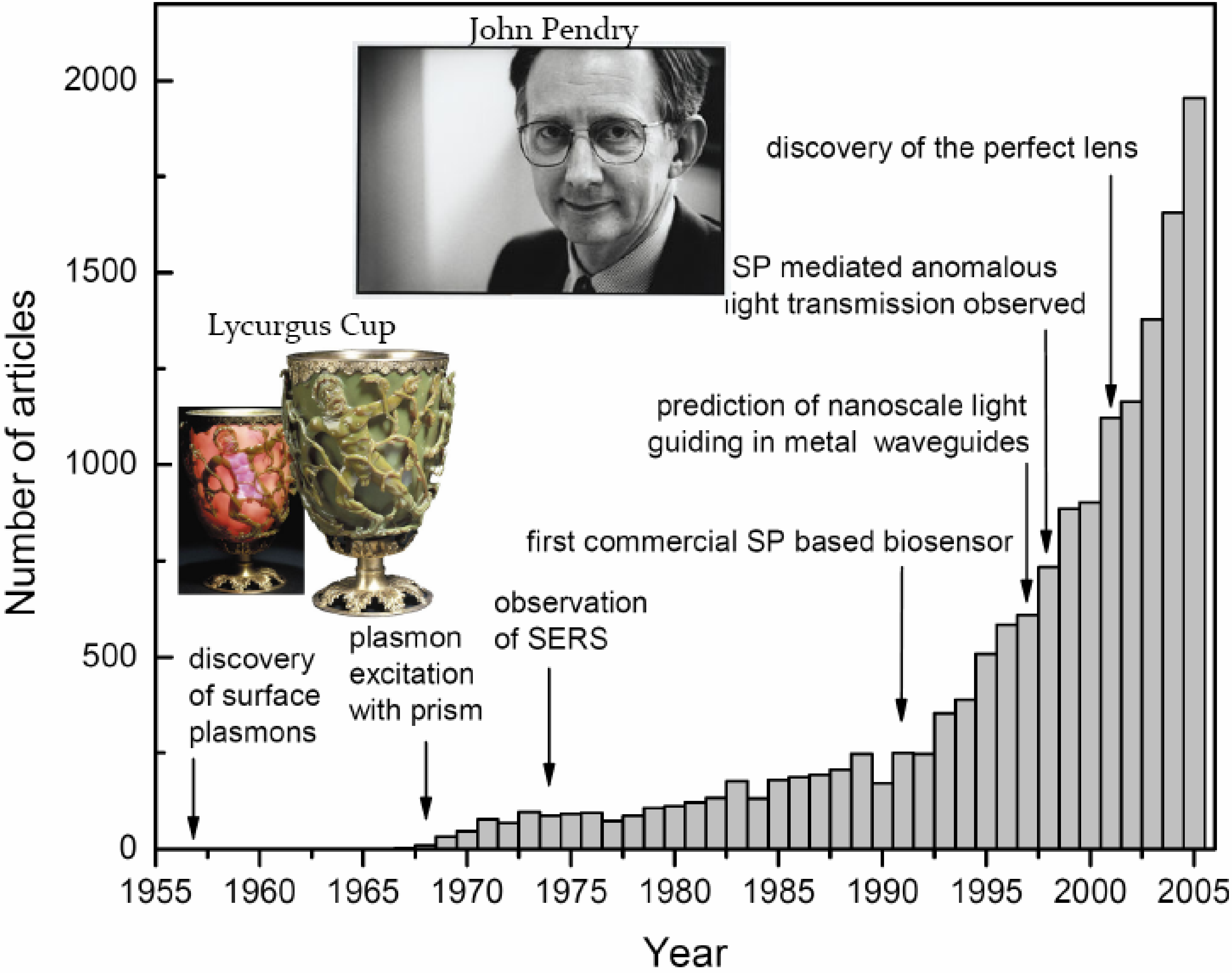
Nature 461, 720-722 (2009).

Ultra-fast modulation using SPP and pumped nonlinear effect



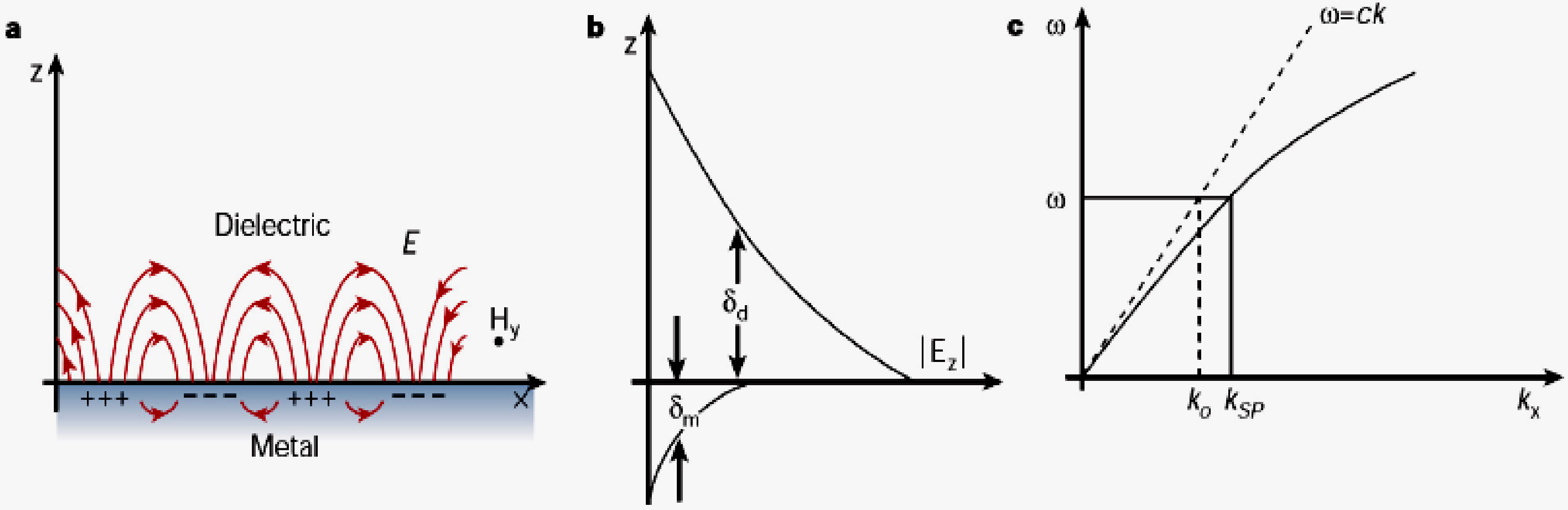
Ultra-fast optical switch

Nature Photonics **3**, 55-58 (2009).



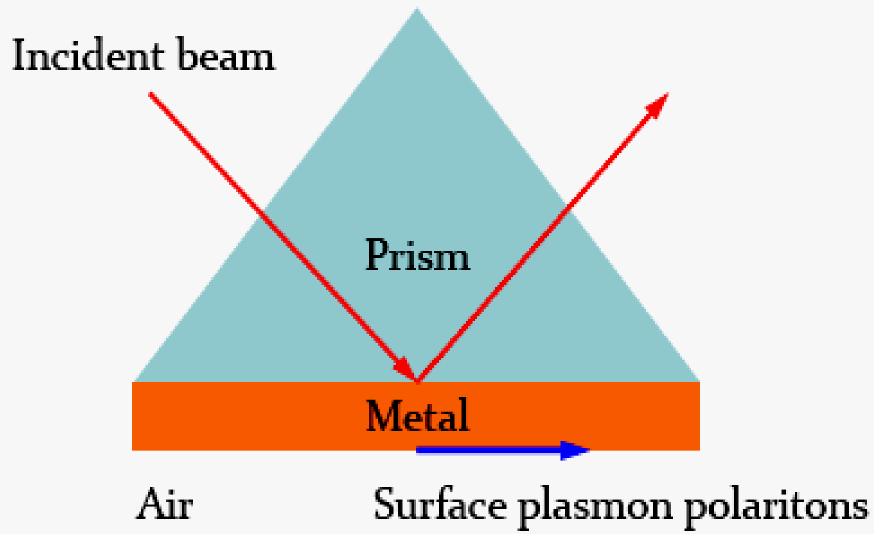
John Pendry
Lycurgus Cup
discovery of the perfect lens
SP mediated anomalous
light transmission observed
prediction of nanoscale light
guiding in metal waveguides
first commercial SP based biosensor
observation
of SERS
plasmon
excitation
with prism
discovery
of surface
plasmons
Number of articles
0
500
1000
1500
2000
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
Year

Surface Plasmon Polaritons



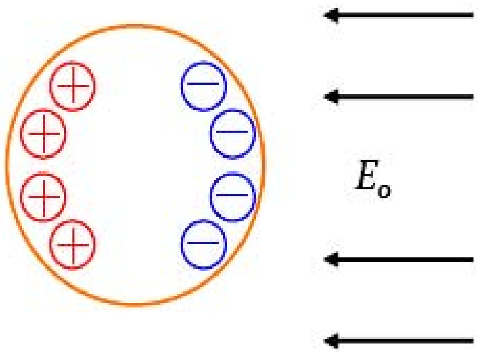
Barnes W.L., Dereux A., Ebbesen T.W., Nature, 2003

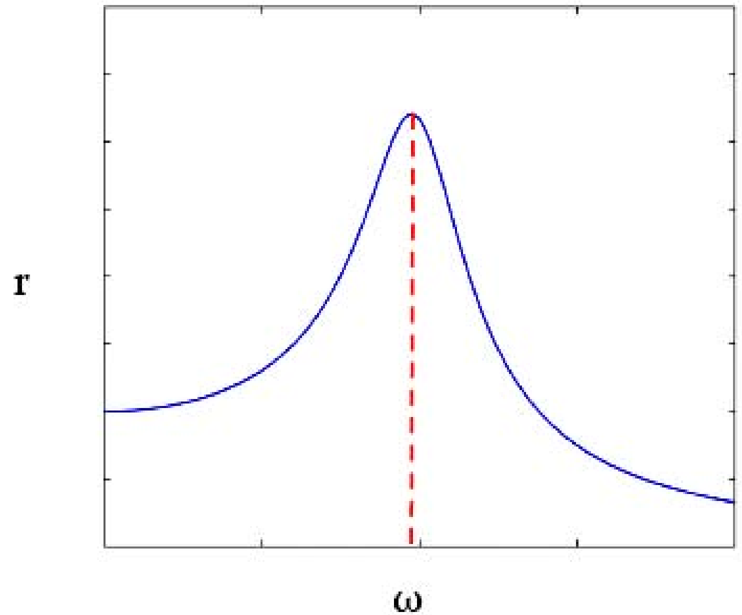
$$k_{sp} = k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}$$



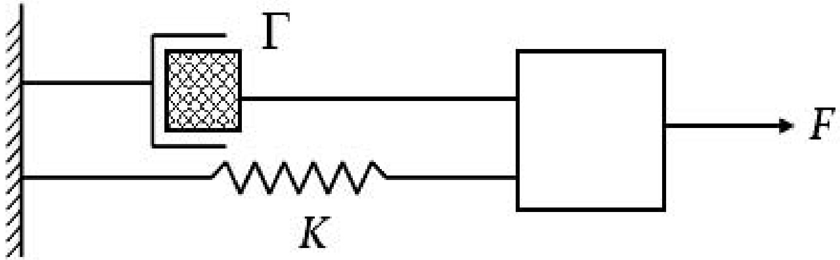
Semi Classic Model for Localized Surface Plasmon

Metal nanoparticle





Driven, damped harmonic oscillator



$$\omega_R = K / m_e^*$$

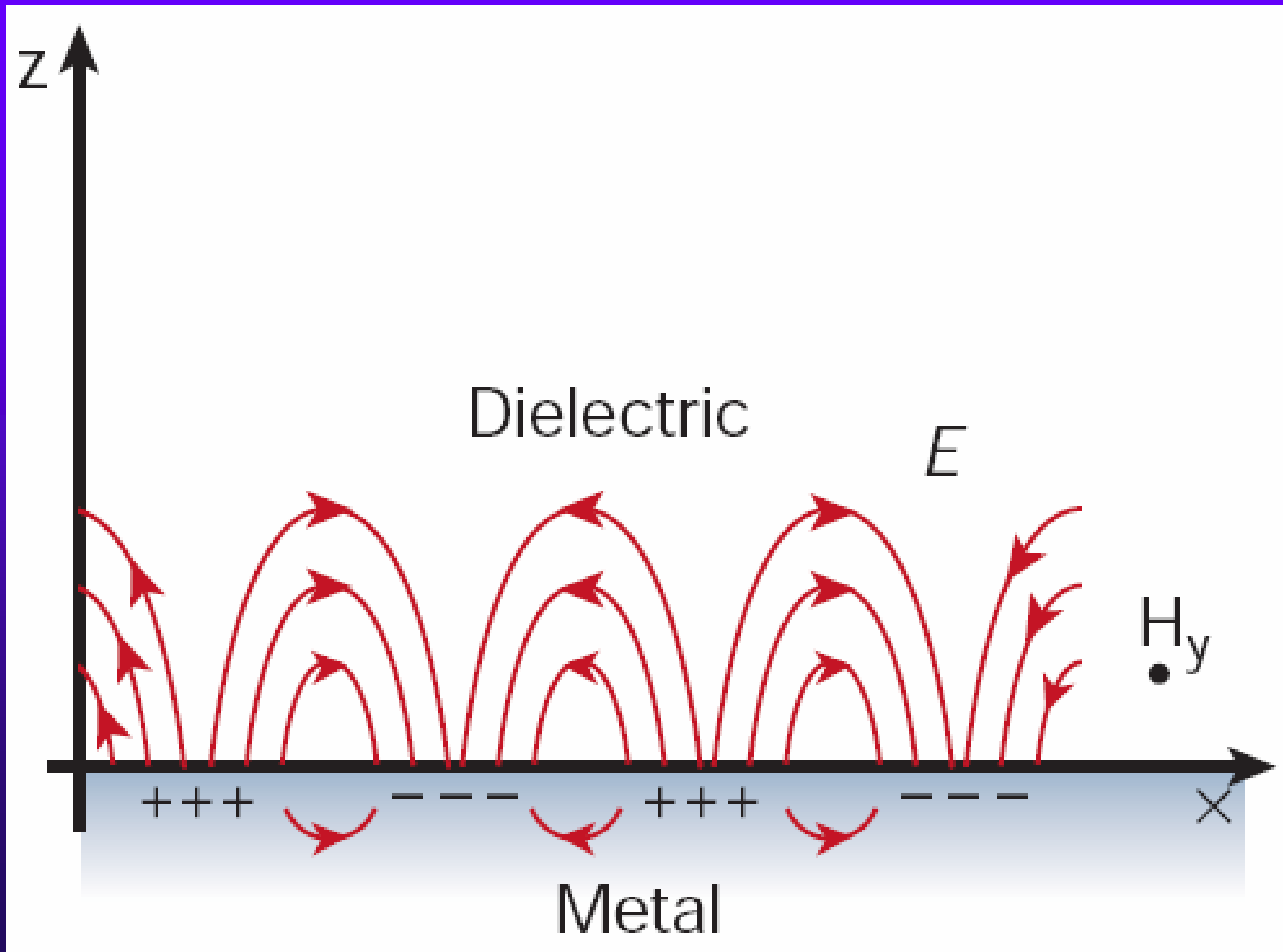
$$FWHM = \sqrt{3}\Gamma$$

$$m_e^* \frac{\partial^2 r}{\partial t^2} + m_e^* \Gamma \frac{\partial r}{\partial t} + Kr = qE_0 e^{-i\omega t}$$

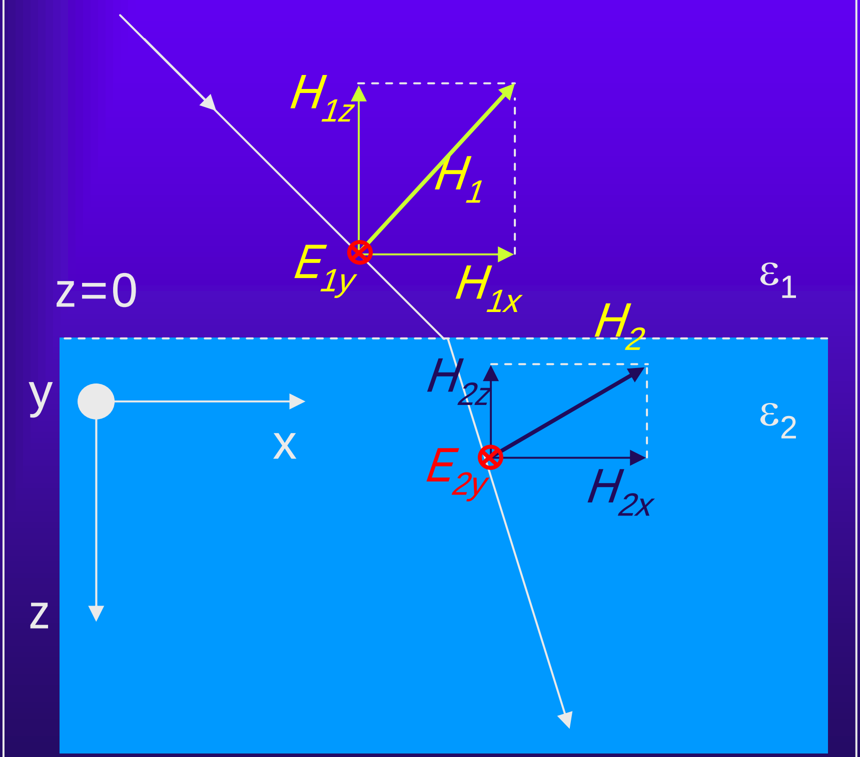
$$r_{\max} = \frac{qE_0}{m_e^* \omega_R \Gamma}$$

✓Polarization charges are created at the interface between two material.

✓The electrons in metal will respond to this polarization giving rise to surface plasmon modes



s-polarized incident radiation does not create polarization charges at the interface. It thus can not excite surface plasmon modes



Boundary condition (note that E-field has a transverse component only):

transverse component of E is conserved,

$$E_{1y} = E_{2y}$$

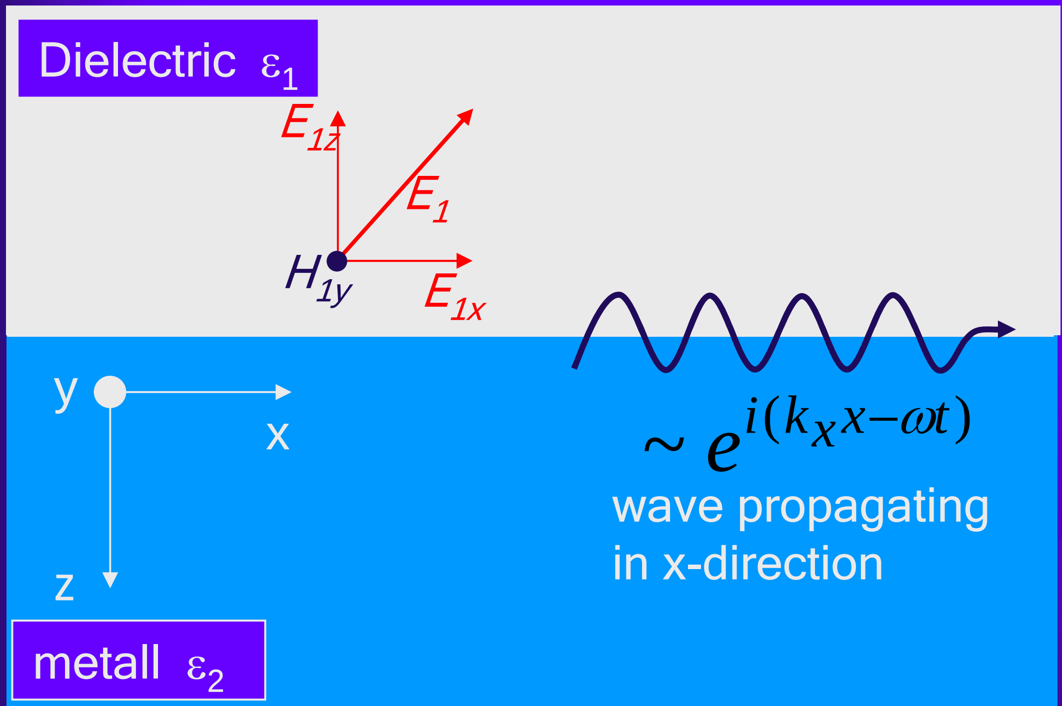
compare with p-polarization:

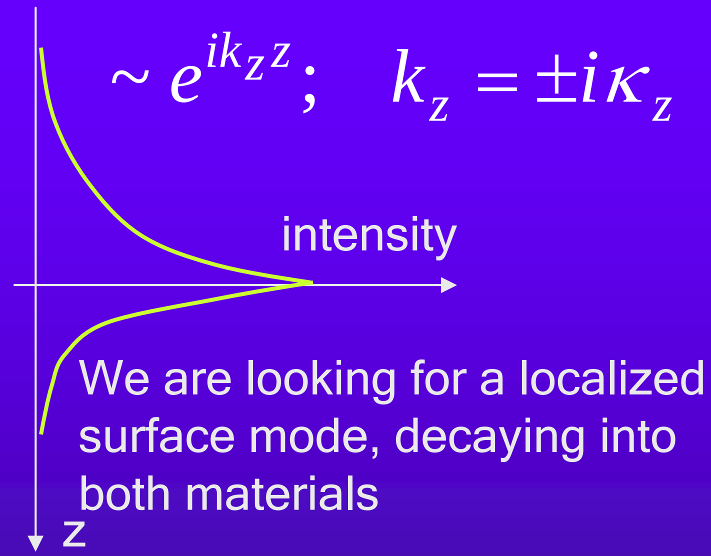
$$\varepsilon_0 E_{0z} + P_{1z} = \varepsilon_0 E_{0z} + P_{2z}$$

no polarization charges are created $\rightarrow$
no surface plasmon modes are excited!
In what follows we shall consider the case of p-polarization only.

More detailed theory

Let us check whether **p-polarized** incident radiation can excite a surface mode





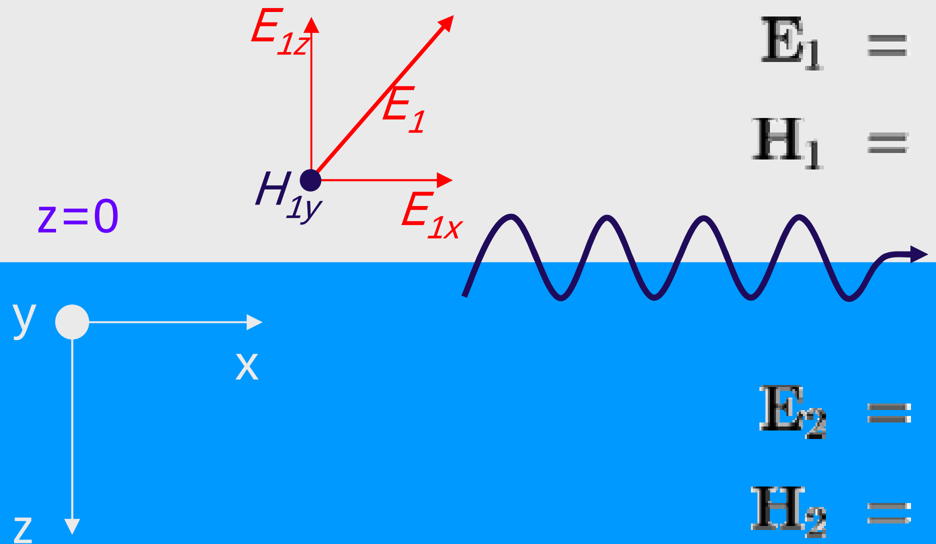
components of E-, H-fields: E = (E_x, 0, E_z); H = (0, H_y, 0)

Thus, the solution can be written as

$$
\begin{aligned}
\mathbf{E} &= (E_x, 0, E_y)\, e^{i(k_x x - \omega t)} e^{ik_z z} \\
\mathbf{H} &= (0, H_y, 0)\, e^{i(k_x x - \omega t)} e^{ik_z z}
\end{aligned}
$$

Solution for a surface plasmon mode:

Dielectric ε_1



$$\mathbf{E}_1 = (E_{1x}, 0, E_{1y})\, e^{i(k_{1x}x-\omega t)} e^{ik_{1z}z}$$

$$\mathbf{H}_1 = (0, H_{1y}, 0)\, e^{i(k_{1x}x-\omega t)} e^{ik_{1z}z}$$

$$\mathbf{E}_2 = (E_{2x}, 0, E_{2y})\, e^{i(k_{2x}x-\omega t)} e^{ik_{2z}z}$$

$$\mathbf{H}_2 = (0, H_{2y}, 0)\, e^{i(k_{2x}x-\omega t)} e^{ik_{2z}z}$$

metall ε_2

Let us see whether this solution satisfies Maxwell equation and the boundary conditions:

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

+

$$E_{1x} = E_{2x}$$

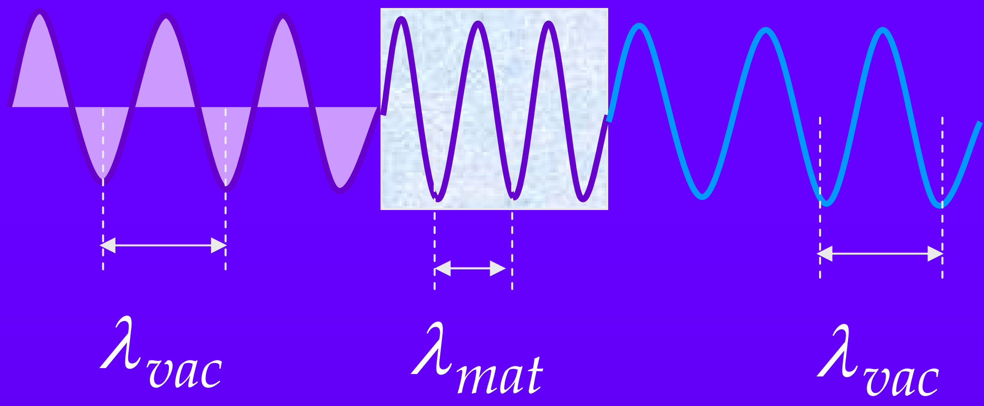
$$H_{y1} = H_{y2}$$

$\Rightarrow$

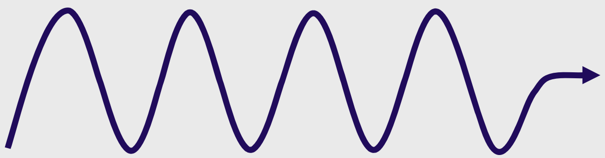
condition imposed on k-vector

$$\frac{\varepsilon_{r1}}{k_{1z}} = \frac{\varepsilon_{r2}}{k_{2z}}$$

$$\lambda_{mat} = \frac{\lambda_{vac}}{n}; \quad n = \sqrt{\varepsilon}$$



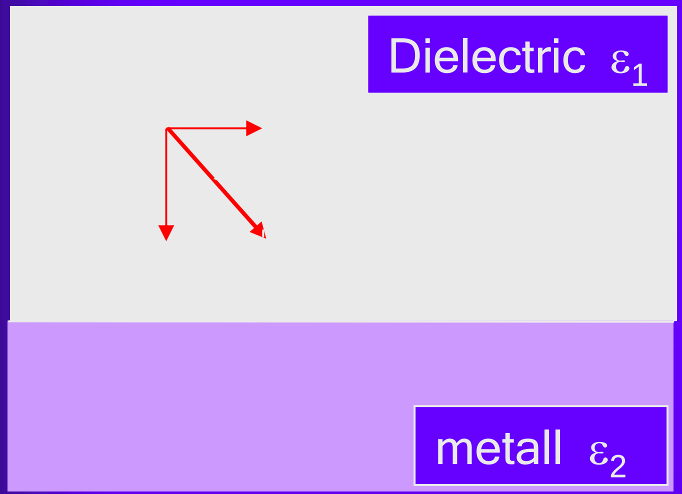
Dielectric n_1



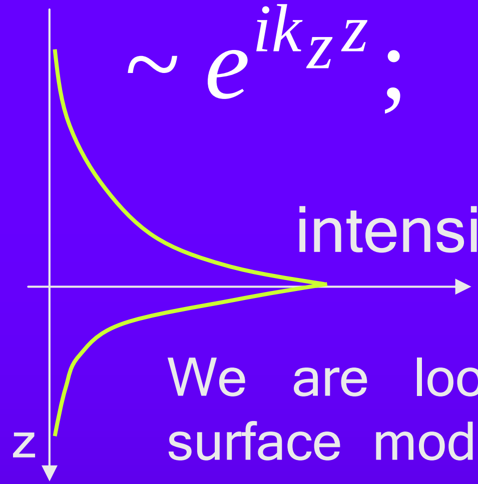
$$\sim e^{i(k_1 x - \omega t)}$$

wave vector in vacuum

$$k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi n_1}{\lambda} = n_1 k_0$$



$$\sim e^{ik_z z}; \quad k_z = \pm i\kappa_z$$



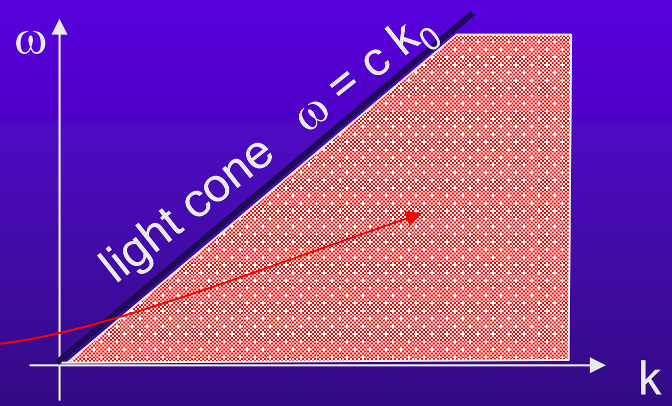
We are looking for a localized surface mode, decaying into both materials → k_z has to be imaginary

$$(n_1 k_0)^2 = k_{1x}^2 + k_{1z}^2$$

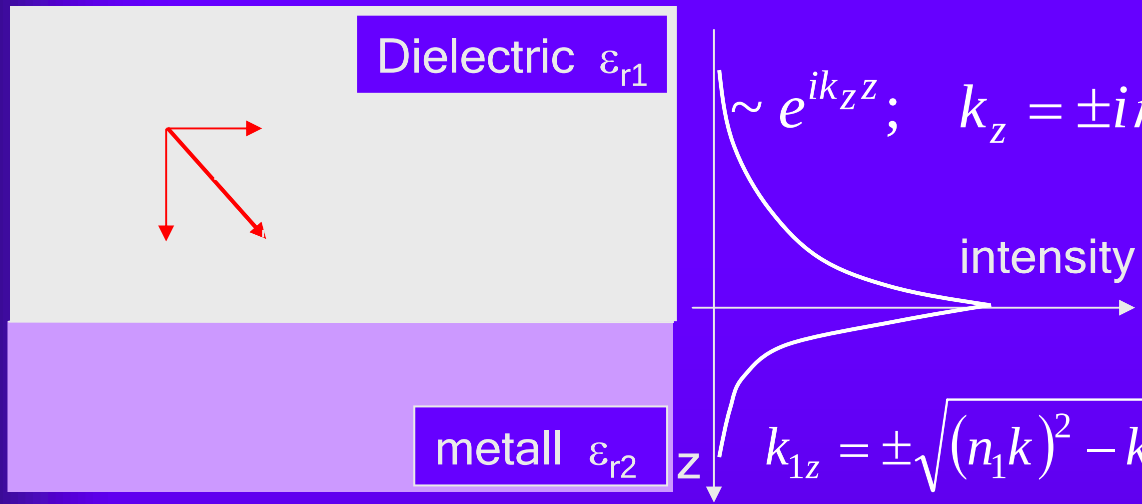
$$k_{1z} = \pm\sqrt{\underbrace{(n_1 k_0)^2 - k_{1x}^2}}$$

$$(n_1 k_0)^2 - k_{1x}^2 < 0$$

$$\boxed{k_{1x} > n_1 k_0}$$



The plasmonic dispersion curve lies beyond the light cone, therefore the direct coupling of propa-gating light to plasmonic states is difficult! ☹



$$\sim e^{ik_z z}; \quad k_z = \pm i\kappa_z$$

$$k_{1z} = \pm\sqrt{(n_1 k)^2 - k_{1x}^2}$$

sign of k_z:

$$k_{1z} = -\sqrt{(n_1 k)^2 - k_{1x}^2}$$

$$k_{2z} = +\sqrt{(n_1 k)^2 - k_{2x}^2}$$

k_{1z} and k_{2z} are of opposite signs!

recall the condition imposed on k-vector:

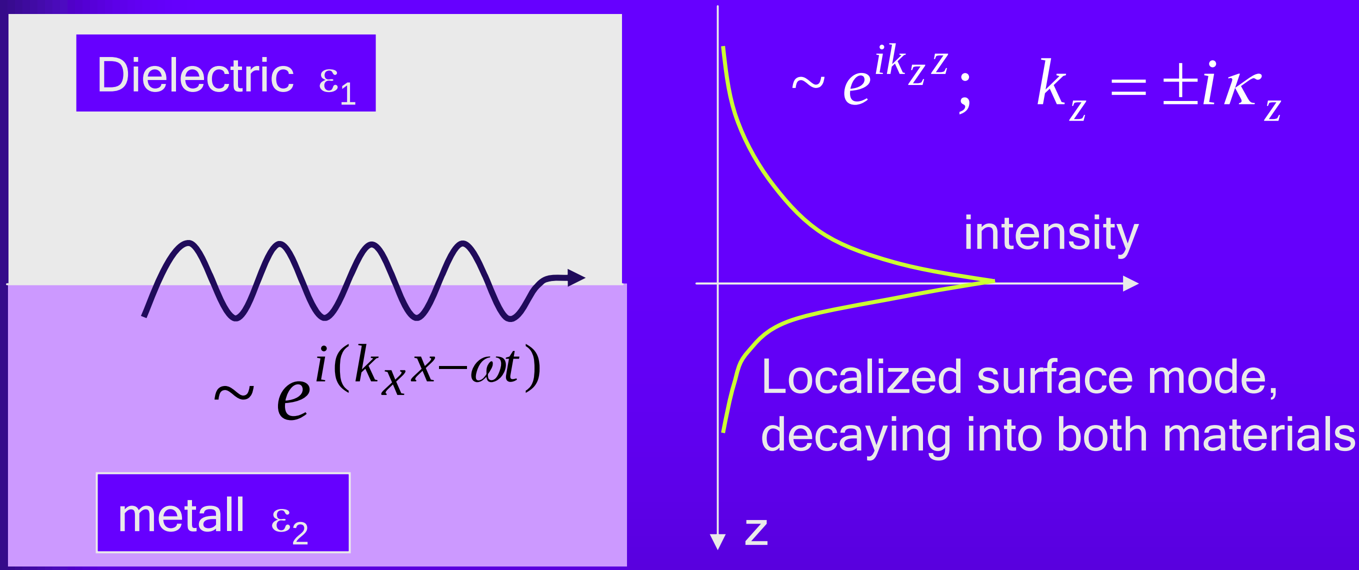
$$\frac{\varepsilon_{r1}}{k_{1z}} = \frac{\varepsilon_{r2}}{k_{2z}}$$

because k_{1z} and k_{2z} are of opposite signs, this condition will be satisfied only if ε_{r1} and ε_{r2} are of opposite signs. This is the case when one material is dielectric $\varepsilon_{r1} > 0$, and the second material is metal, $\varepsilon_{r1} < 0$.

also, recall the condition

$$k_{2x}^2 > (n_2 k)^2 = \varepsilon_{r2} k^2$$

this condition is always satisfied for metals, where $\varepsilon_{r2} < 0$



$$\mathbf{E} = (E_x, 0, E_y)\, e^{i(k_x x - \omega t)} e^{ik_z z}$$
$$\mathbf{H} = (0, H_y, 0)\, e^{i(k_x x - \omega t)} e^{ik_z z}$$

Thus, we have established that on the surface between a metal and dielectric one can excite a localized surface mode. This localized mode is called a surface plasmon

What is the wavelength of the surface plasmon $\lambda = \dfrac{2\pi}{k}$?

let us find *k:*

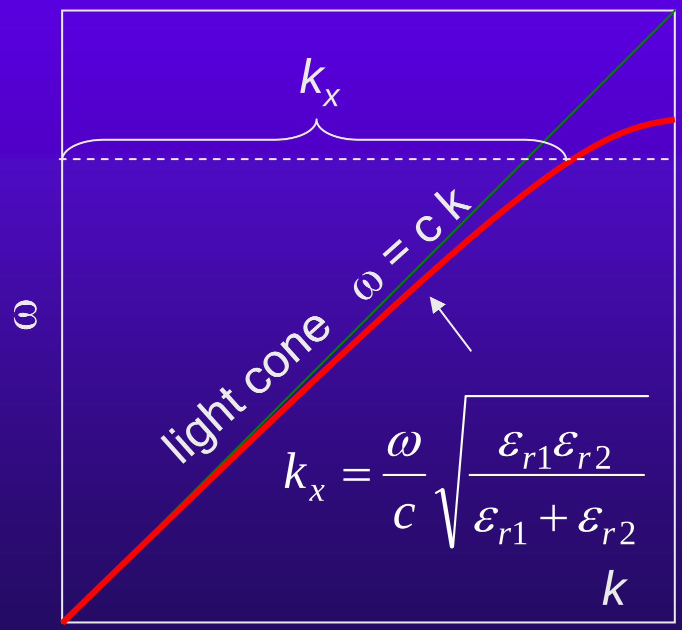
$$\frac{\varepsilon_{r1}}{k_{1z}} = \frac{\varepsilon_{r2}}{k_{2z}}$$

substitute

$$k_{1z} = -\sqrt{(n_1 k)^2 - k_{1x}^2}$$

$$k_{2z} = +\sqrt{(n_1 k)^2 - k_{2x}^2}$$

$$\boxed{k_x = k\sqrt{\frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}}}$$



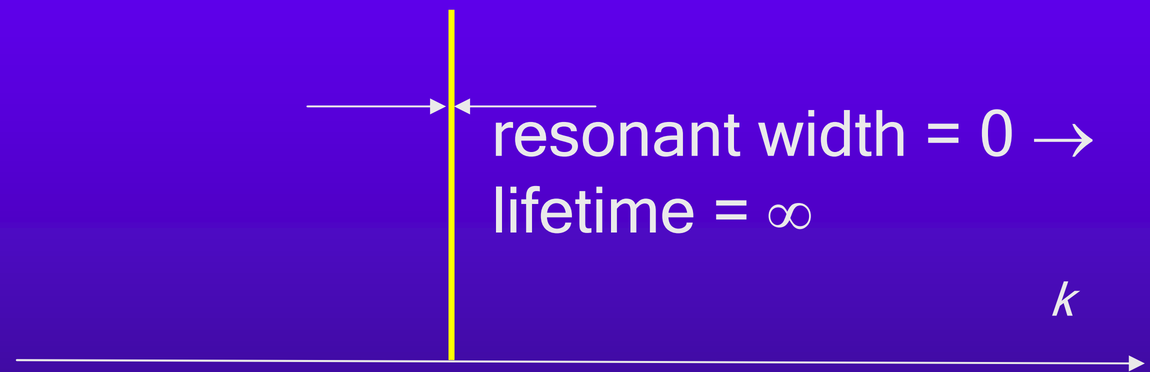
The surface plasmone mode always lies beyond the light line, that is it has greater momentum than a free photon of the same frequency ω

Ideal case: ε_{r1} and ε_{r2} are real (no imaginary components = no losses)

Dielectric: $\varepsilon_{r1} > 0$

Metal: $\varepsilon_{r2} < 0$, $|\varepsilon_{r2}| >> \varepsilon_{r1}$

$\left.\begin{array}{l} \\ \end{array}\right\} k_x$ is real



$$k_x = k_0 \sqrt{\frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1} + \varepsilon_{r2}}}$$

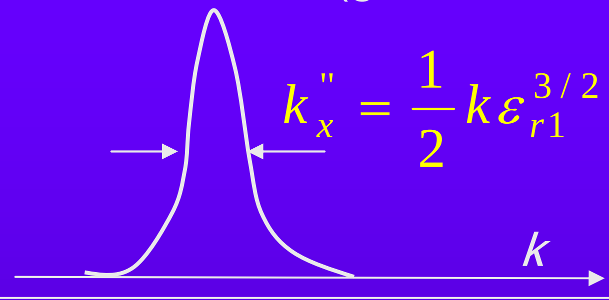
Realistic case: ε_{r1} is real, and ε_{r2} is complex,

$$\varepsilon_{r2} = \varepsilon'_{r2} + i\varepsilon''_{r2} \longleftarrow$$ imaginary part describes losses in metal

$$k_x = k\sqrt{\frac{\varepsilon_{r1}\varepsilon_{r2}}{\varepsilon_{r1}+\varepsilon_{r2}}} = k\sqrt{\frac{\varepsilon_{r1}\left(\varepsilon'_{r2}+i\varepsilon''_{r2}\right)}{\varepsilon_{r1}+\left(\varepsilon'_{r2}+i\varepsilon''_{r2}\right)}}$$

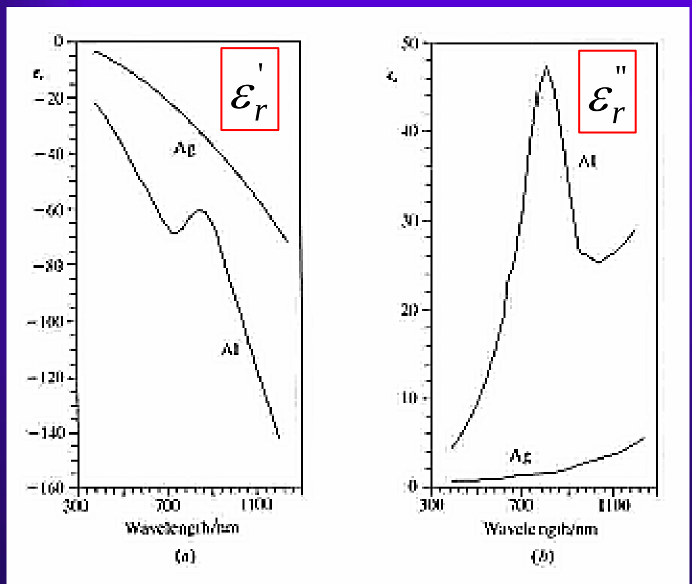
$$= \ldots = k'_x + ik''_x$$

resonant width (gives rise to losses)

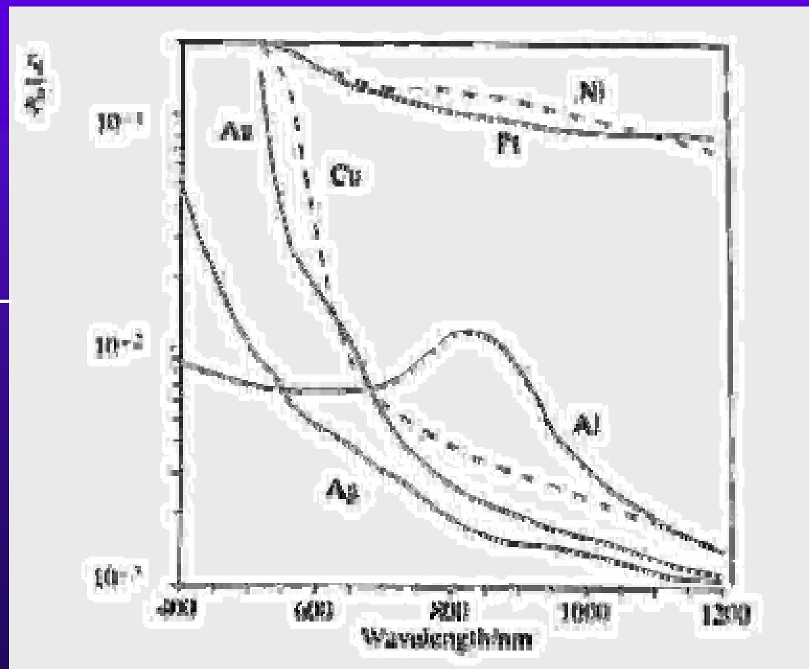


$$k''_x = \frac{1}{2}k\varepsilon_{r1}^{3/2}\frac{\varepsilon''_{r2}}{\left(\varepsilon'_{r2}\right)^2}$$

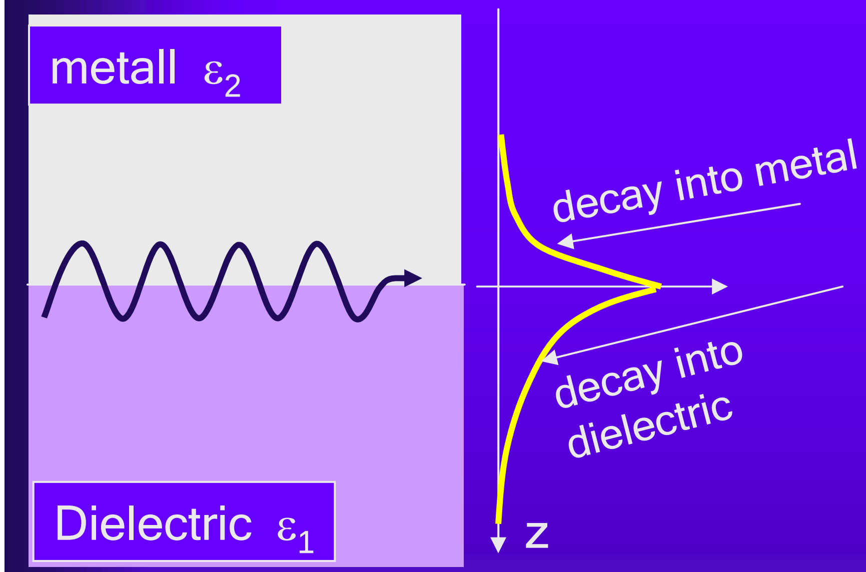
Dielectric functions of Ag, Al

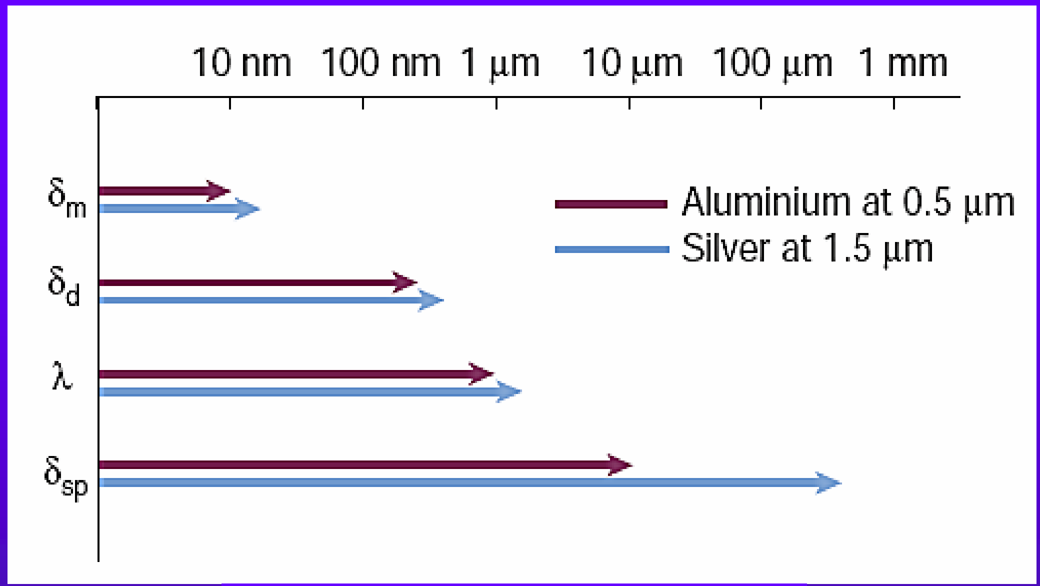


$$\frac{\varepsilon''_{r2}}{\left(\varepsilon'_{r2}\right)^2}$$



surface plasmon length scales:

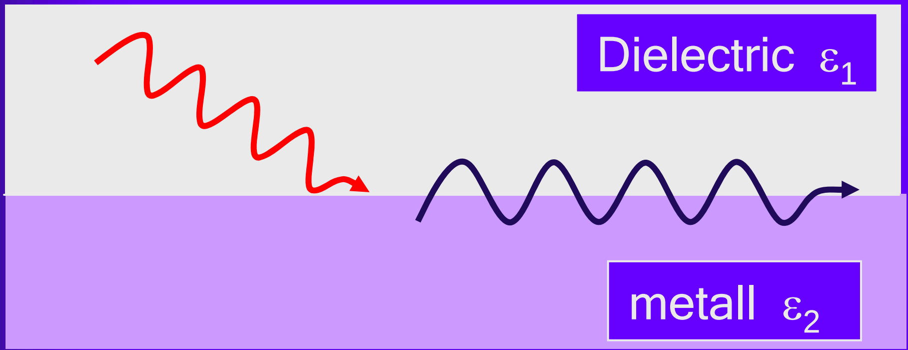


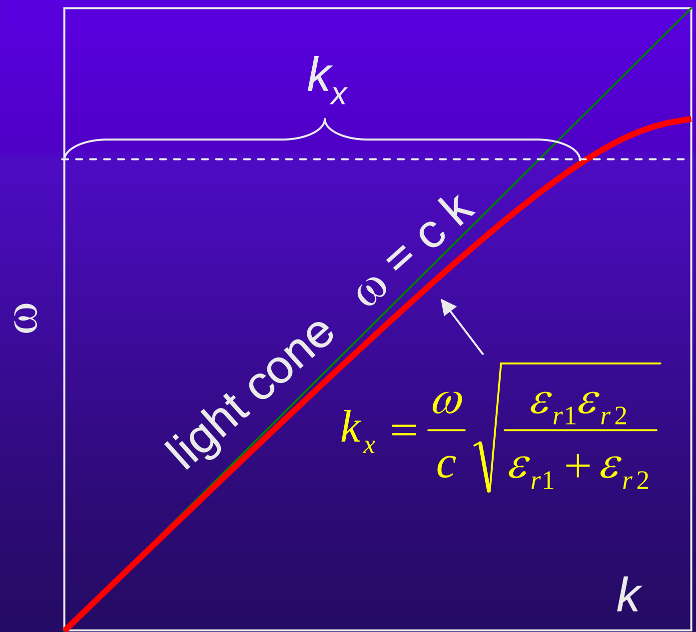


propagation length

How to excite a surface plasmon?

Is it possible to excite a plasmon mode by shining light on a dielectric/metal interface?



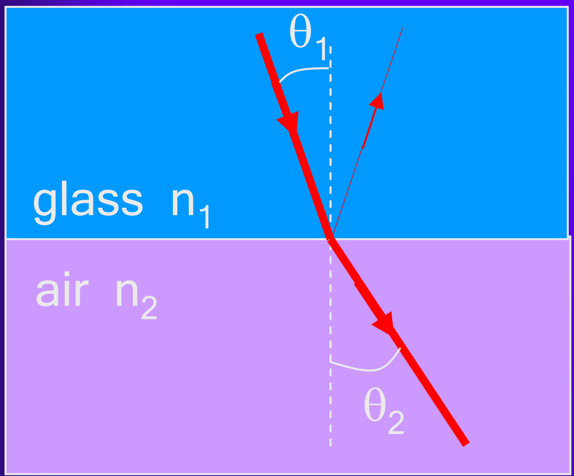


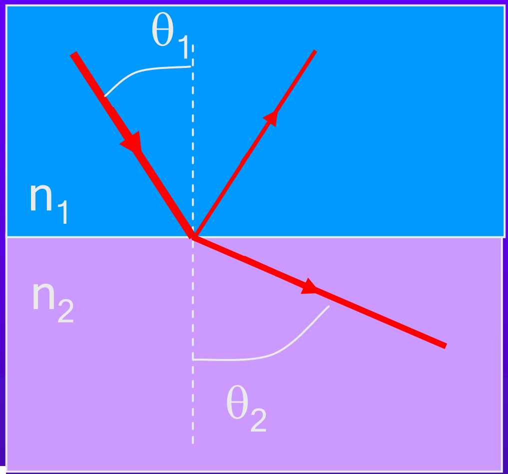
The surface plasmone mode always lie beyond the light line. It has greater momentum than a free photon of the same frequency ω.

This makes a direct excitation of a surface plasmon mode impossible!

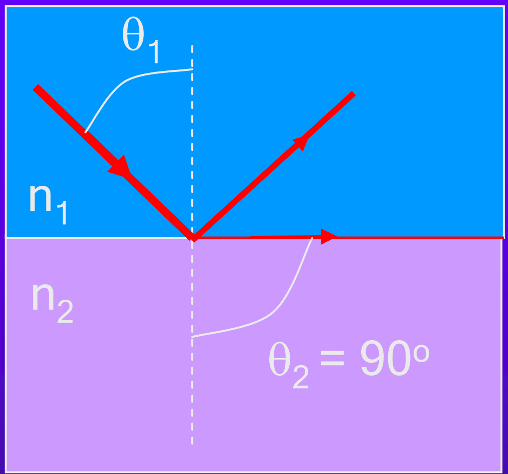
Total internal reflection

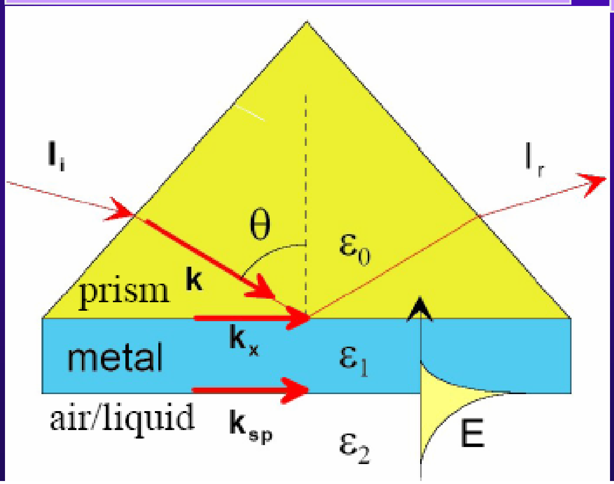
Snell's law of refraction: $n_2 \sin\theta_2 = n_1 \sin\theta_1$ $\quad n_2 < n_1 \Rightarrow \dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{n_1}{n_2} > 1$





Total internal reflection



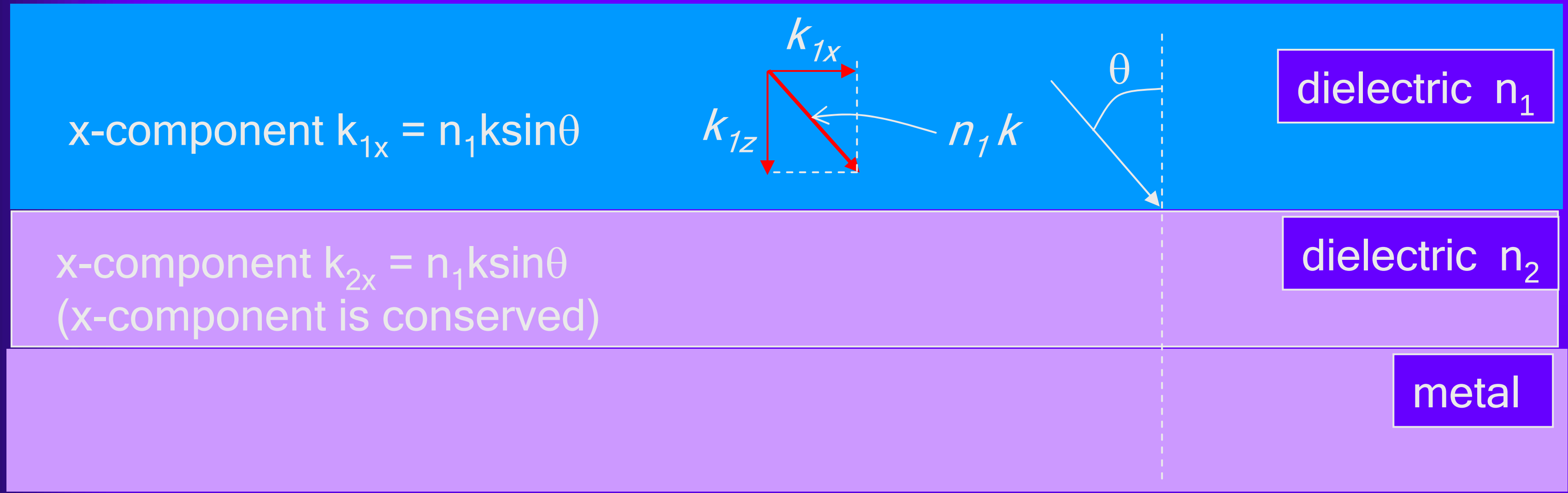


critical angle of the
total internal reflection:

$$\theta_2 = 90^0 \Rightarrow$$

$$\sin\theta_c = \frac{n_2 \sin 90^0}{n_1} = \frac{n_2}{n_1}$$

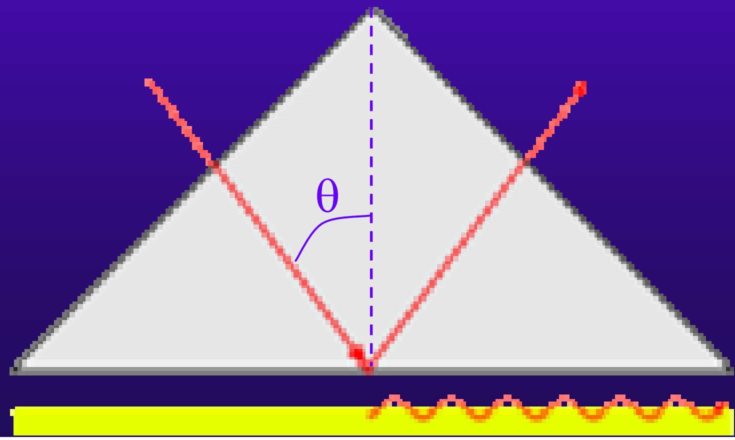
Otto geometry



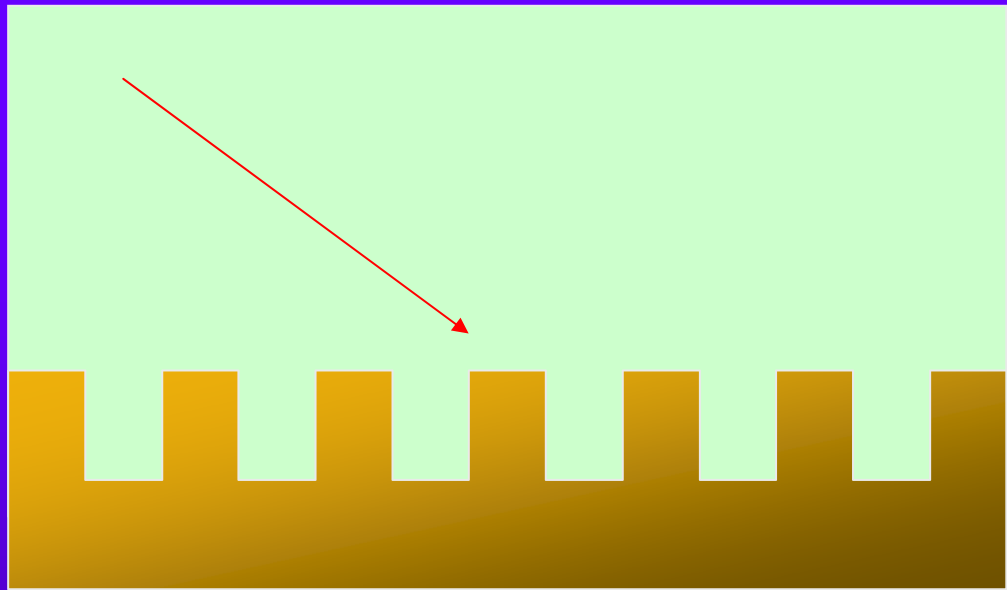
to excite a plasmon mode in the region 2:

$k_{2x} > n_2 k$

$$n_1 k \sin\theta > n_2 k \quad \Rightarrow \quad \sin\theta > \frac{n_2}{n_1}$$ condition for the total internal reflection!



Utilization of a grating to excite a plasmon mode



Grating

The grooves in the grating surface break the translation invariance and allow k_x of the outgoing wave to be different from that of the incoming wave

$$\underbrace{k_x\ \text{(outgoing)}}_{k_{\text{plasmon}}} = \underbrace{k_x\ \text{(incoming)}}_{nk\sin\theta} \pm \underbrace{NG,}_{\text{reciprocal lattice vectors}} \quad \text{where } G = 2\pi/d$$

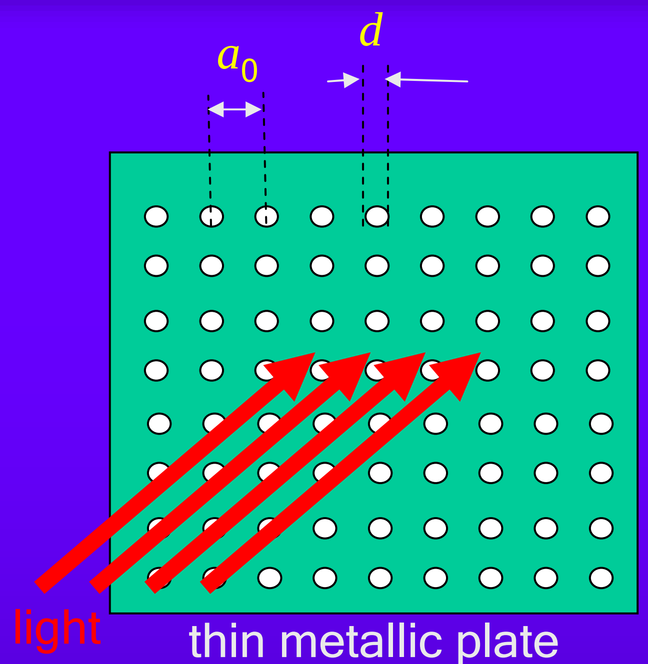
APPLICATION OF SURFACE PLASMONS

- *Extraordinary transmission through sub-wavelength hole arrays*, T. W. Ebbesen *et al.,* Nature 391, 667 (1998).
- *Directional beaming*, H. J. Lezec *et al.,* Science 297, 820 (2002)
- *Plasmonic nanowire waveguides*, J. B. Kren *et al.,* Europhys. Lett. 60, 663 (2002)
- *Nanofocusing in plasmonic waveguides,* M. Stockman, Phys. Rev. Lett. 93, 137404 (2004).
- *Nanoparticle plasmon waveguide*, S. A. Maier *et al*., Nature Materials 2, 229 (2003).
- *Surface plasmon enhanced solar cells*

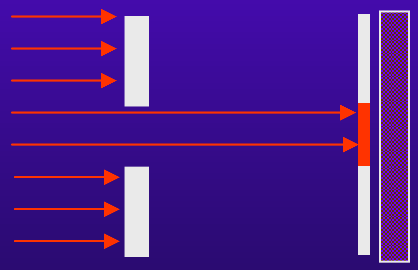
Extraordinary optical transmission through sub-wavelength hole arrays

T. W. Ebbesen*†, H. J. Lezec‡, H. F. Ghaemi*, T. Thio* & P. A. Wolff*§

NATURE | VOL 391 | 12 FEBRUARY 1998

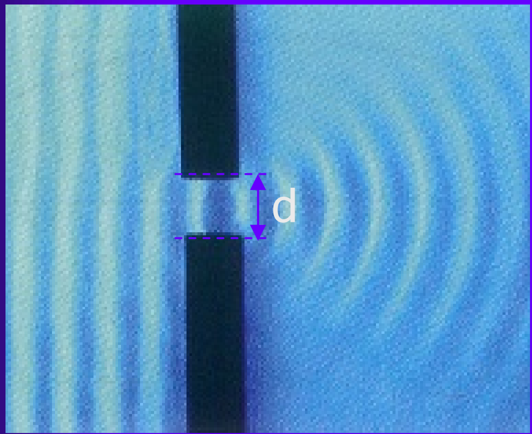


classical (ray optics) expectation:

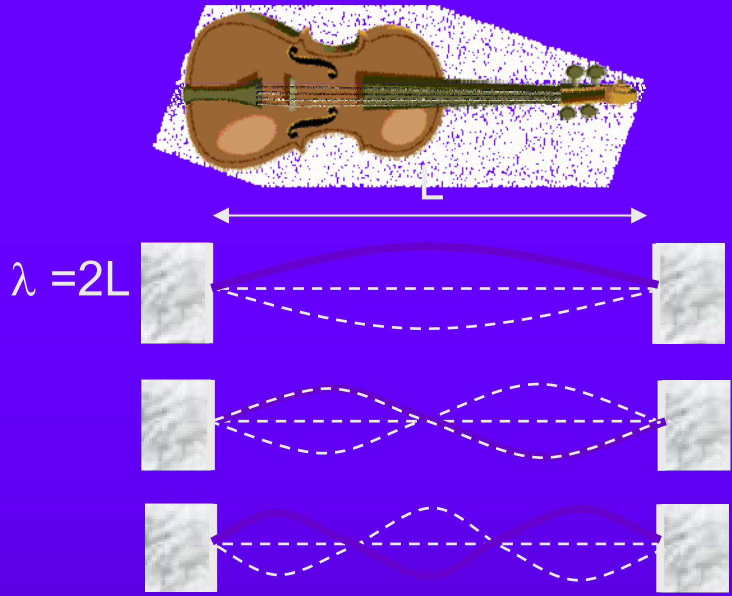


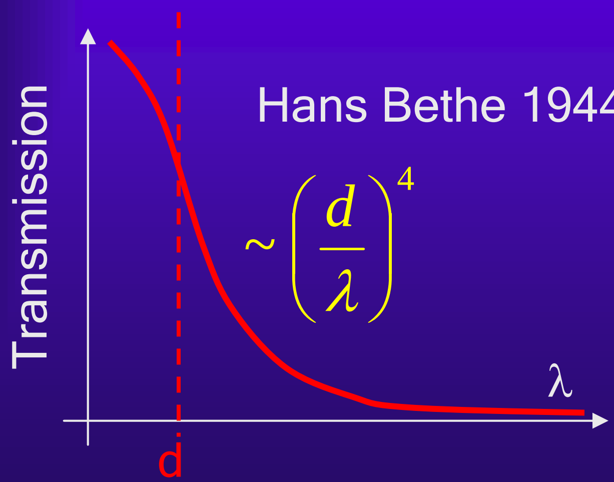
$$\text{transmission} = \frac{\text{area of the plate}}{\text{area occupied by holes}}$$

wave optics: diffraction effect



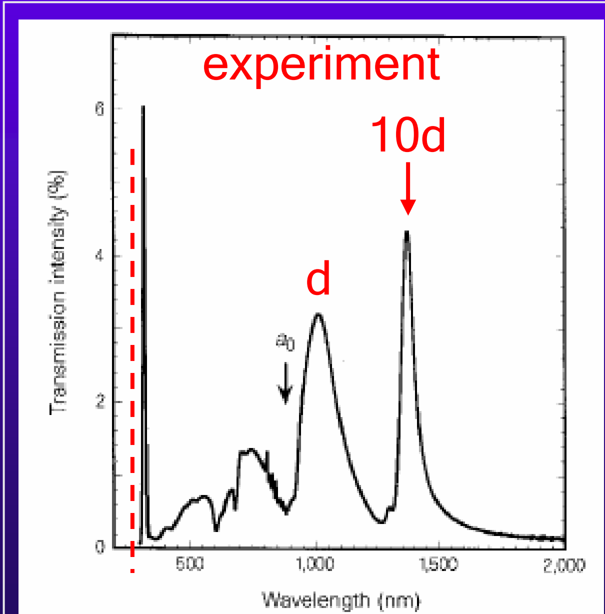
if λ/2 > d, the transmission through the hole will be strongly suppressed



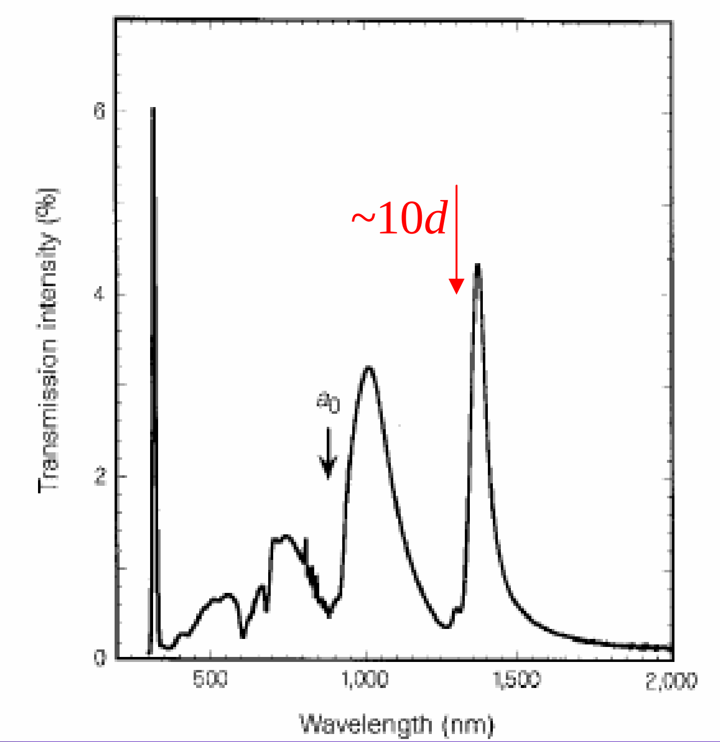


Hans Bethe 1944

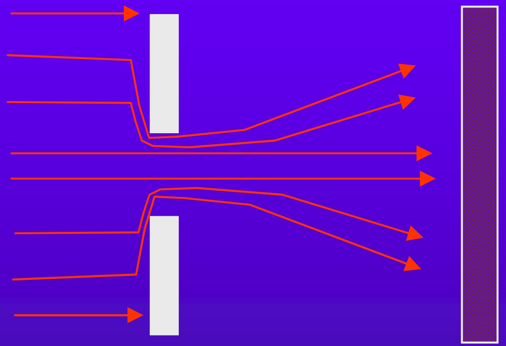
$$\sim \left(\frac{d}{\lambda}\right)^4$$



The experimental findings imply that that the array itself is an active element, not just a passive geometrical object in the path of incident light absolute transmission intensity = $\dfrac{\text{transmitted light}}{\text{fraction of area occupied by the holes}}$ = 200%



This observation implies that the light impinging on the metal between holes can be transmitted. In other words, the whole structure acts like an antenna



Explanation: all the observed features are related to excitation of the surface plasmonics. [No enhanced transmission is observed for semiconductor hole arrays.] The resonant peaks occur when surface plasmon momentum matches the momentum of the incident photon and the grating as follows

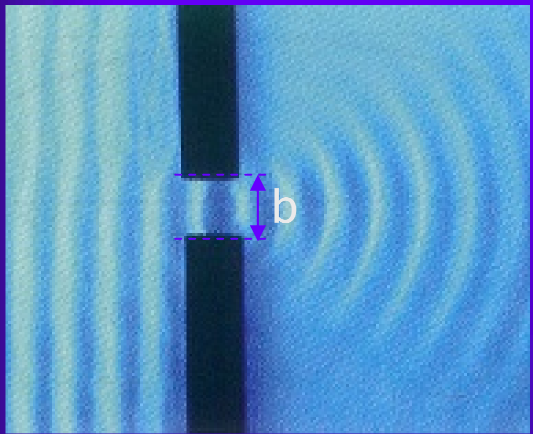
$k_{sp} = k_x \pm nG_x \pm mG_y$ $G_x = G_y = 2\pi/a_0$ are the grating momentum

Beaming Light from a Subwavelength Aperture

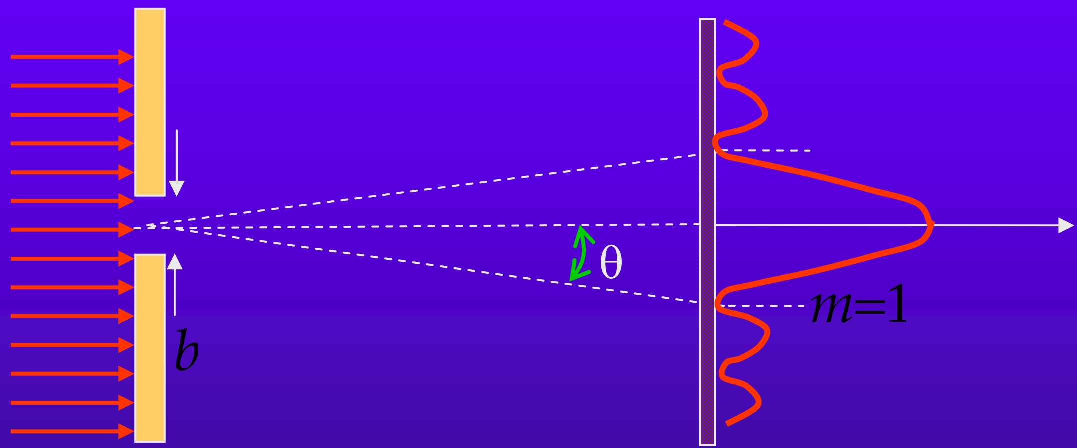
H. J. Lezec,[1] A. Degiron,[1] E. Devaux,[1] R. A. Linke,[2] L. Martin-Moreno,[3] F. J. Garcia-Vidal,[4] T. W. Ebbesen[1*]

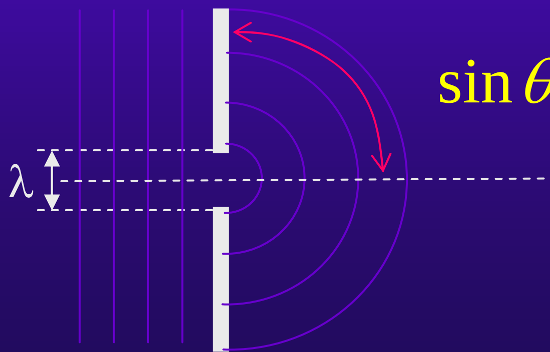
2 AUGUST 2002 VOL 297 SCIENCE

Standard diffraction theory: diffraction on a slit:



position of the central maximum: $\sin\theta = \dfrac{\lambda}{b}$



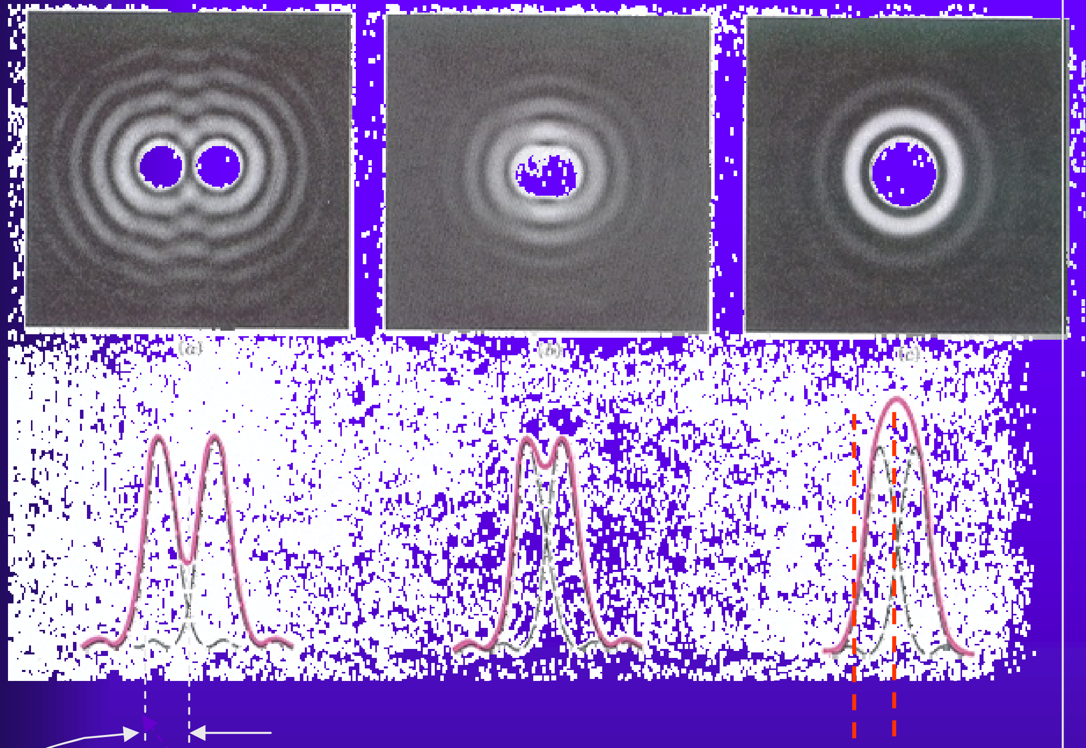


$$\sin\theta = 1 \quad \Rightarrow \quad \theta = 90^\circ$$

diffraction puts a lower limit on the size of the feature that can be used in photonics.

Resolving power is given by the diffraction limit.

diffraction pattern of two point sources



first minimum

$$\theta \sim \frac{\lambda}{a}$$

$$\theta_c \sim \frac{\lambda}{a}$$

first minimum coincides with the maximum

To increase resolution people usually use smaller wavelength

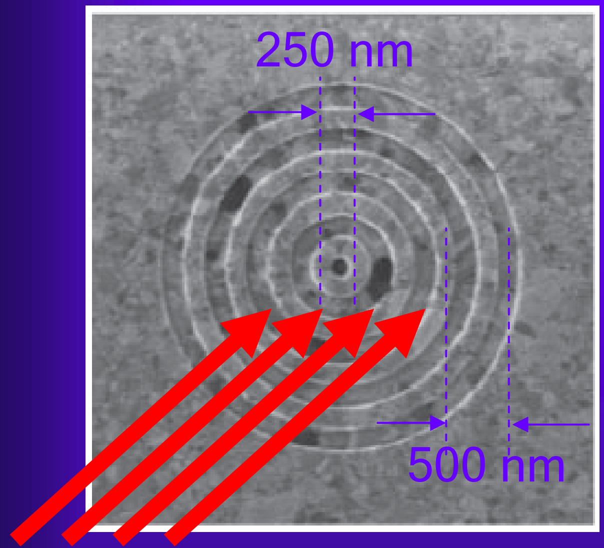
Transmission electron microscope

$\lambda_{el} = 3.7 \times 10^{-3}$ nm

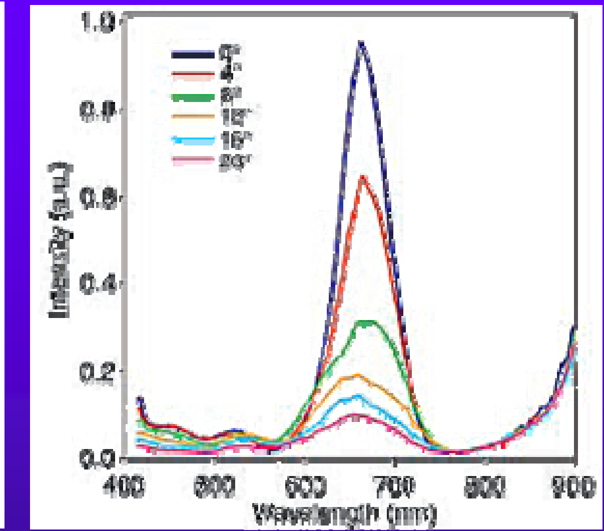
resolution: 1nm

Overcoming the diffraction limit with the help of surface plasmons

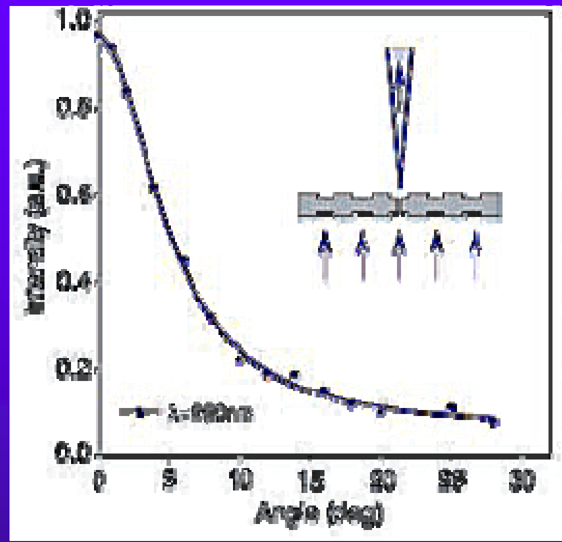
metallic (Ag) film



surface plasmon resonance @ 660nm

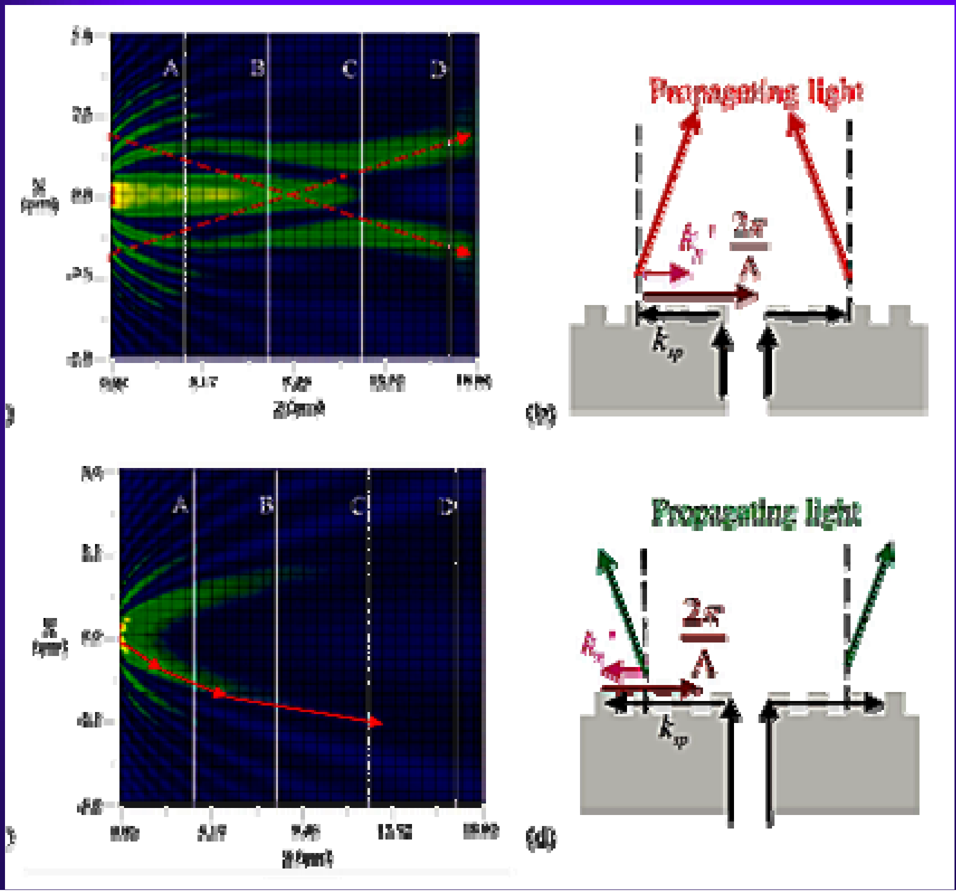


directionality is $\pm$ 3°



The coupling of light into SP modes is governed by geometrical momentum, selection rules (*i.e.*, occurs only at a specific angle for a given wavelength), the light exiting a single aperture will follow the reverse process in the presence of the periodic structure on the exit surface.

$$\underbrace{k_x\,(\text{outgoing})}_{k_{\text{plasmon}}} = \underbrace{k_x\,(\text{incoming})}_{nk\sin\theta} \pm \underbrace{NG}_{\text{reciprocal lattice vectors}},\quad \text{where } G = 2\pi/d$$



$k_{SP}<2\pi/\Lambda$

$k_{SP}=2\pi/\Lambda$?

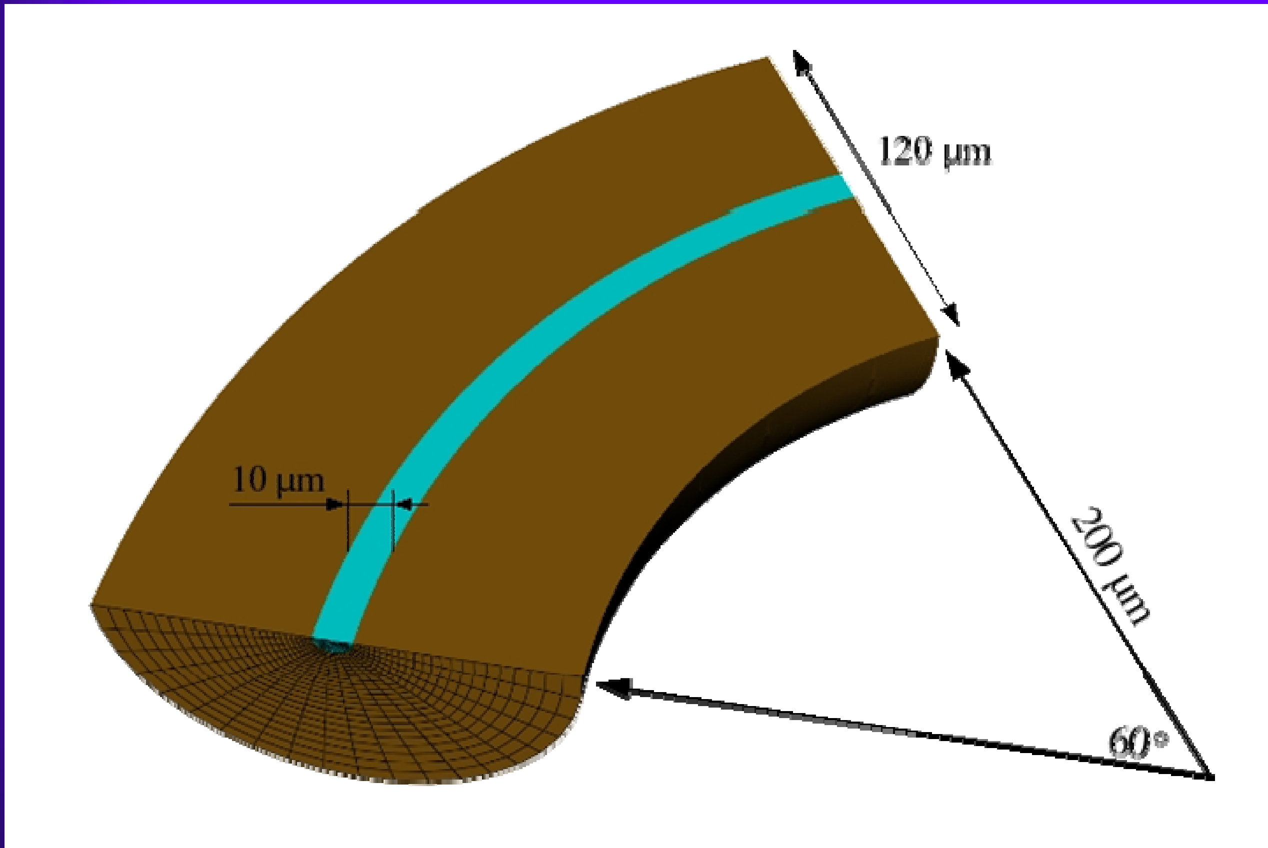
$k_{SP}>2\pi/\Lambda$

Yu et al., Phys. Rev. B 71, 041405 (R) (2005)

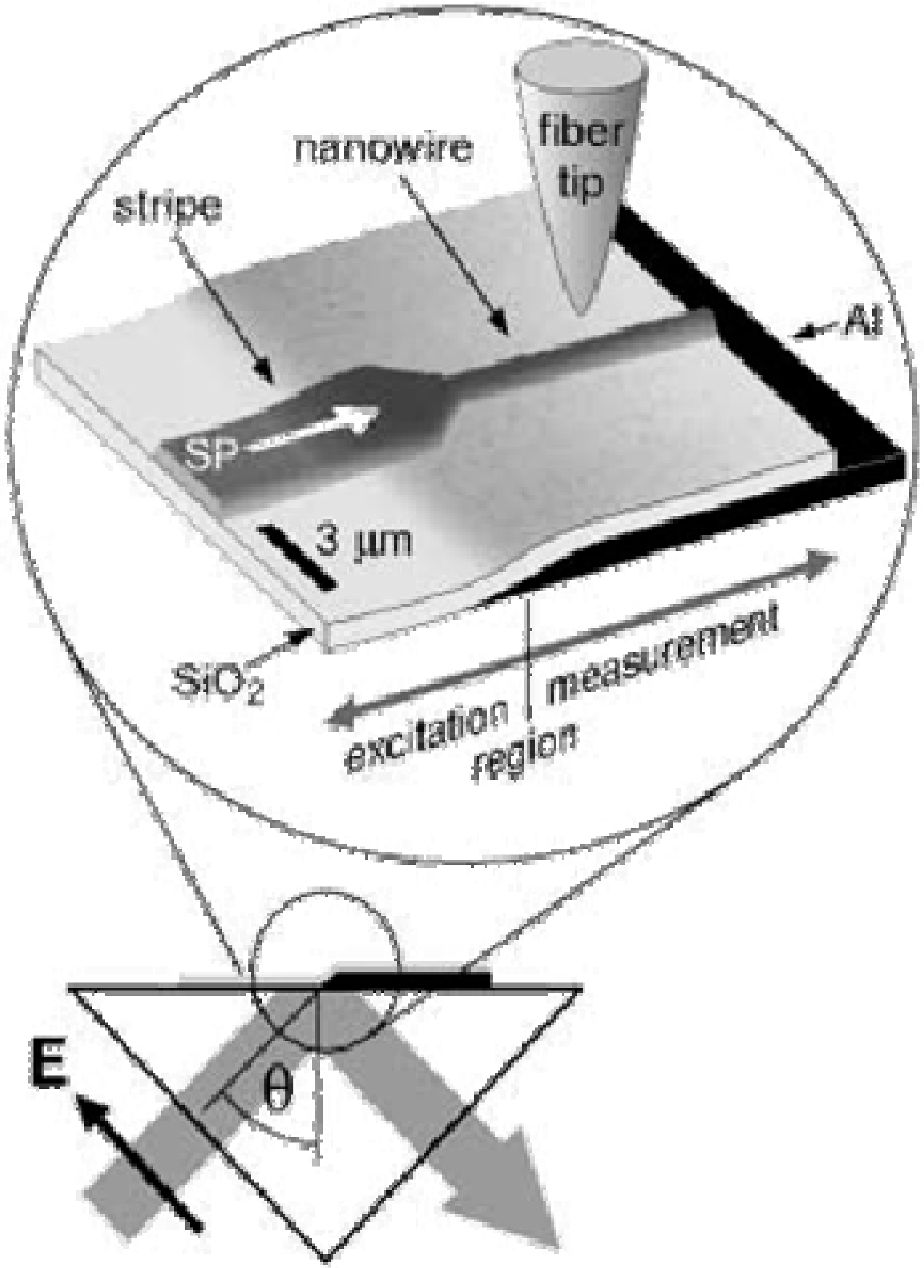
Non–diffraction-limited light transport by gold nanowires

J. R. Krenn, B. Lamprecht, H. Ditlbacher, G. Schider,
M. Salerno, A. Leitner and F. R. Aussenegg

EUROPHYSICS LETTERS
Europhys. Lett., 60 (5), pp. 663–669 (2002)

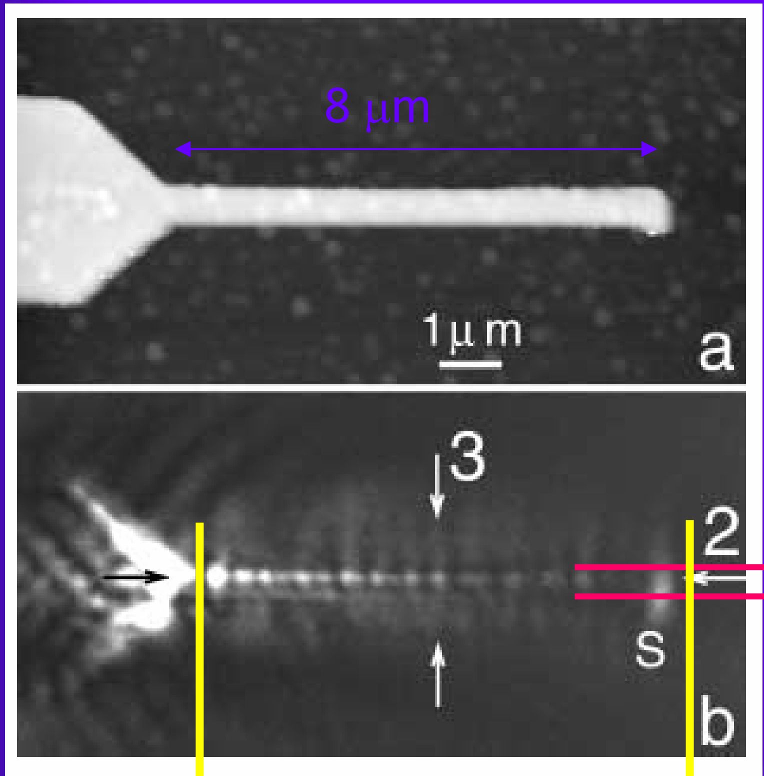


The miniaturization of dielectric waveguides is limited by diffraction to dimensions of the order of the wavelength in the waveguide core.

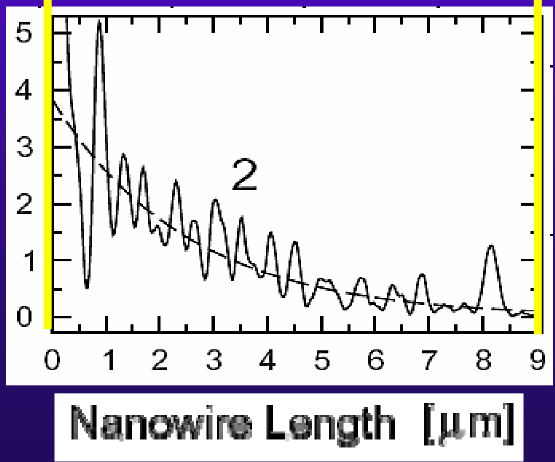


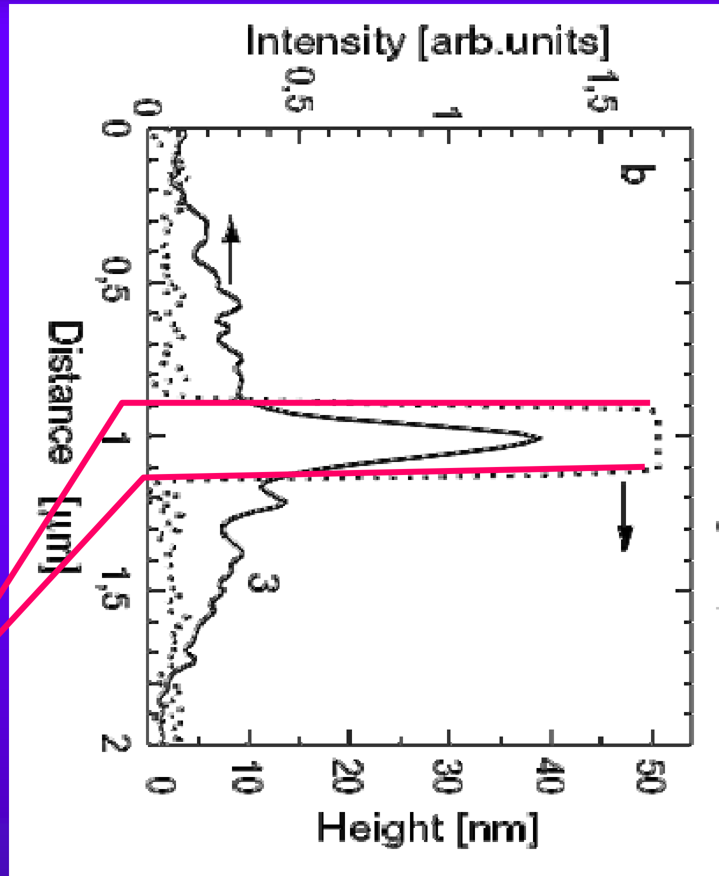
Metal nanowires sustaining surface plasmons can be used as optical waveguides. Thereby, the use of a metal allows to overcome the limitations of miniaturization imposed on conventional dielectric waveguides due to diffraction.

STM microphotography of the device



Optical near-field intensity





It has been shown that the diffraction limit restricting the further miniaturization of dielectric waveguides can be broken by using gold nanowires to guide light fields via surface plasmons excitation. A propagation length of 2.5 μm was found for gold nanowires.

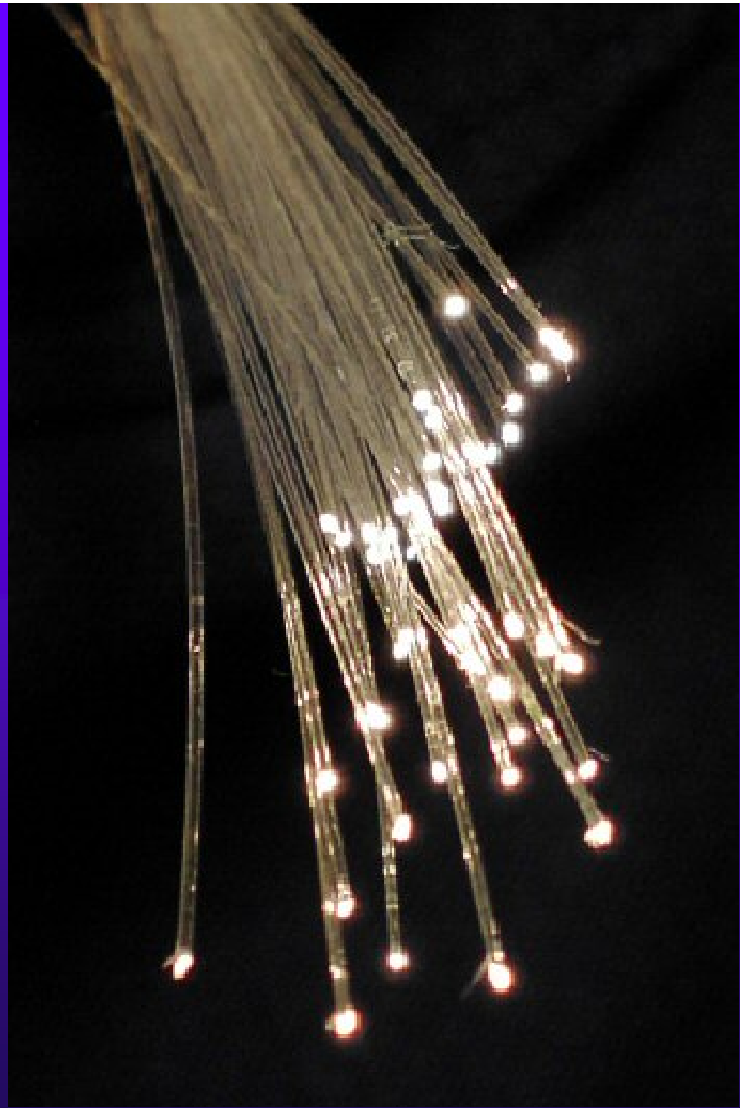
VOLUME 93, NUMBER 13 PHYSICAL REVIEW LETTERS week ending 24 SEPTEMBER 2004

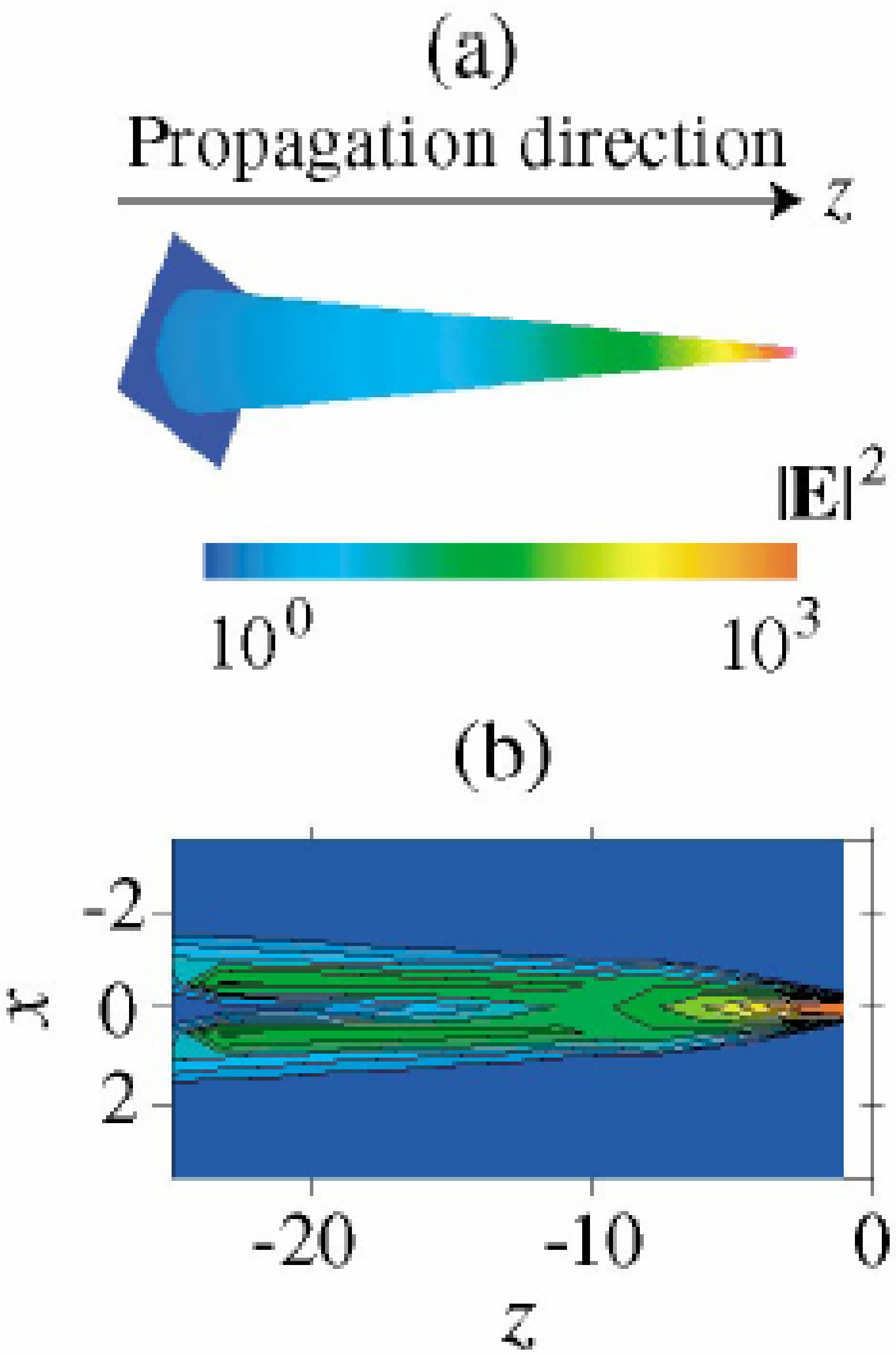
Nanofocusing of Optical Energy in Tapered Plasmonic Waveguides

Mark I. Stockman

The central problem of the nano-optics is the delivery and concentration (nanofocusing) of the optical radiation energy on the nanoscale,

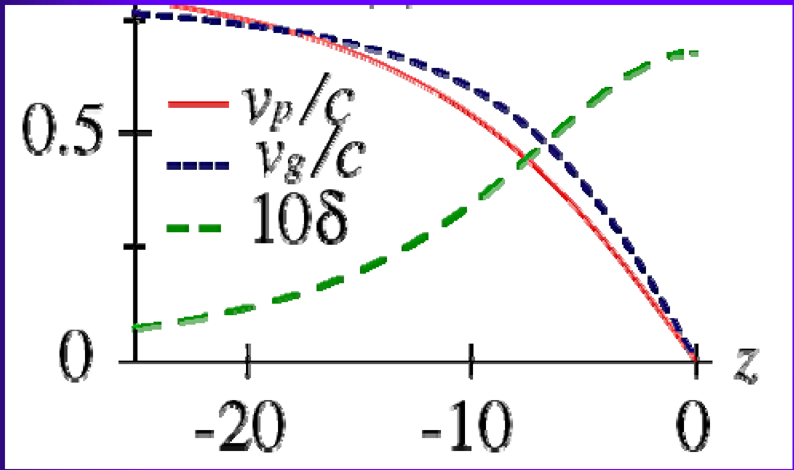
This represents a difficult task, because the wavelength of light is on the microscale, many orders of magnitude too large.

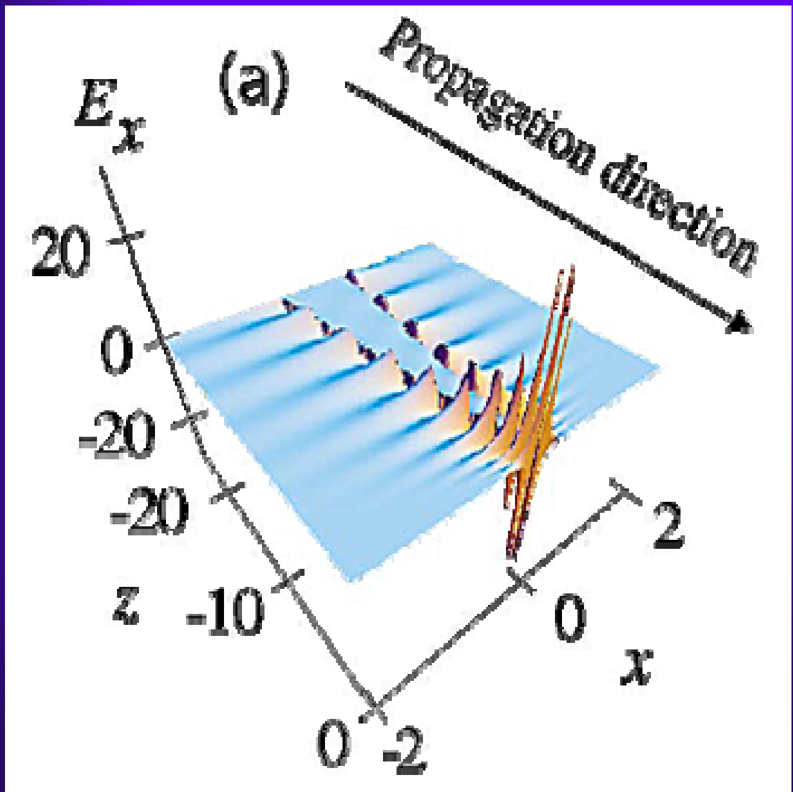




It was shown show that it is possible to focus and concentrate in three dimensions the optical radiation energy on the nanoscale without major losses.

This can be done by exciting the surface plasmons propagating toward a tip of a tapered metal-nanowire surface-plasmonic waveguide.





Both the phase and group velocity of surface plasmons asymptotically tend to zero toward the nanotip. Consequently, the surface plasmons are slowed down and adiabatically stopped at $z = 0$.

This phenomenon leads to a giant concentration of energy on the nanoscale.

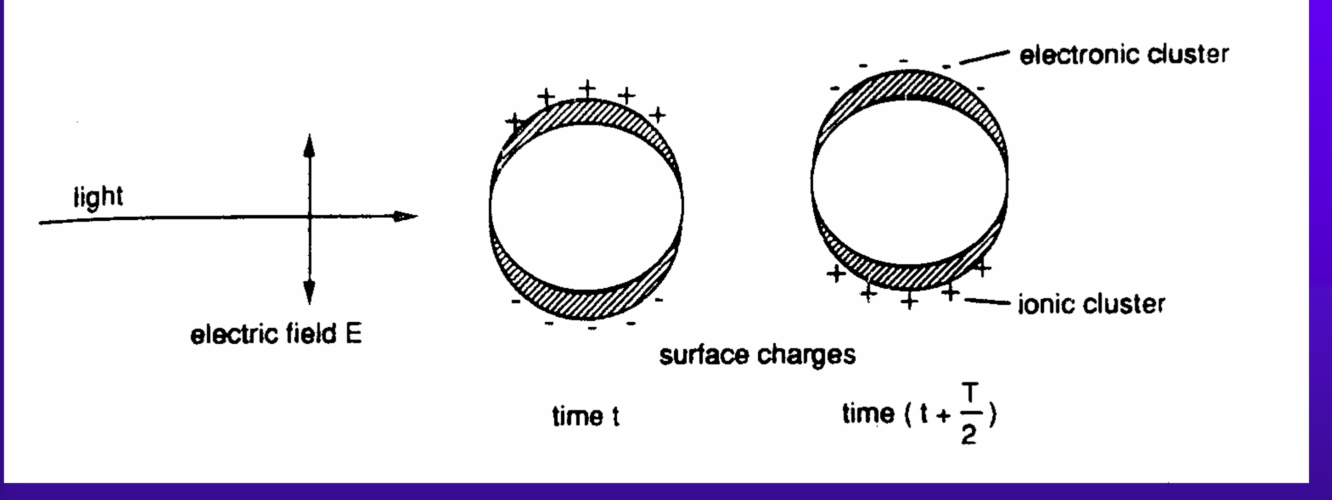
The local field increase by 3 orders of magnitude in intensity and four orders in energy density.

Local detection of electromagnetic energy transport below the diffraction limit in metal nanoparticle plasmon waveguides

STEFAN A. MAIER*[1], PIETER G. KIK[1], HARRY A. ATWATER[1], SHEFFER MELTZER[2], ELAD HAREL[2], BRUCE E. KOEL[2] AND ARI A.G. REQUICHA[2]

nature materials | VOL 2 | APRIL 2003 | www.nature.com/naturematerials

metallic particles support plasmonic resonances:



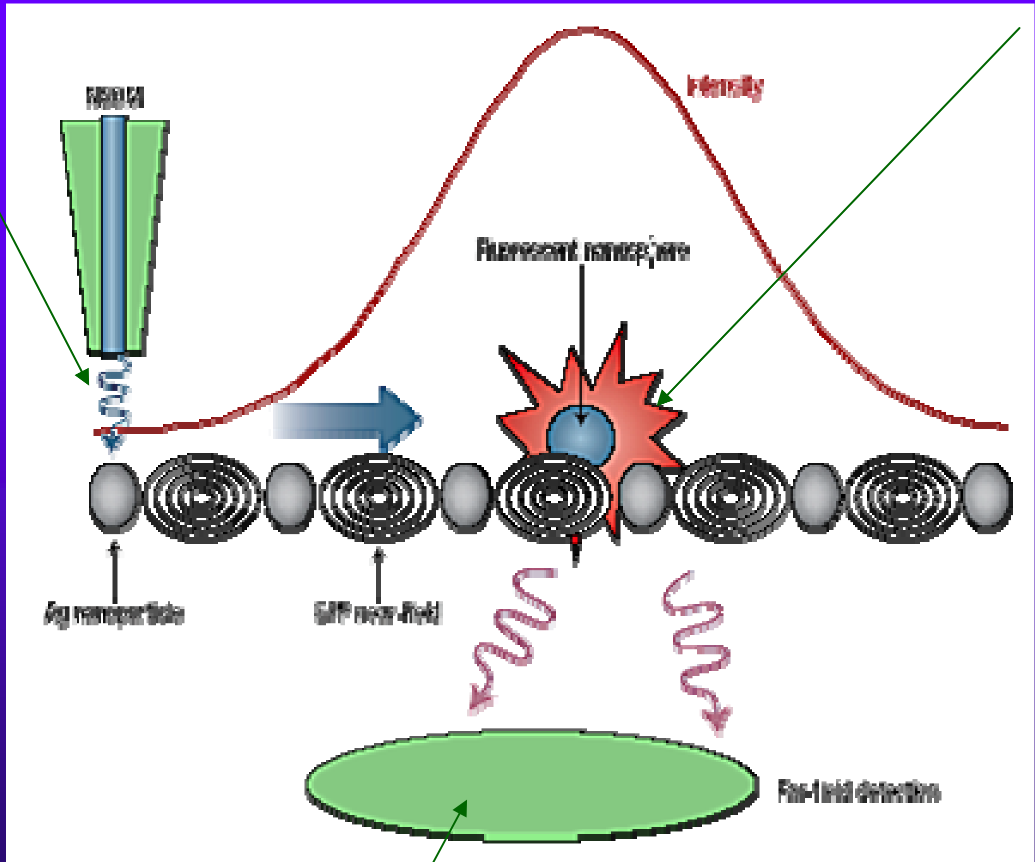
condition for plasmonic resonance:

metall → $\varepsilon'(\omega) = -2\varepsilon_m$ ← host dielectric material

Watching energy transfer: Excitation and detection of energy transport in metal nanoparticle chains by near-field optical microscopy.

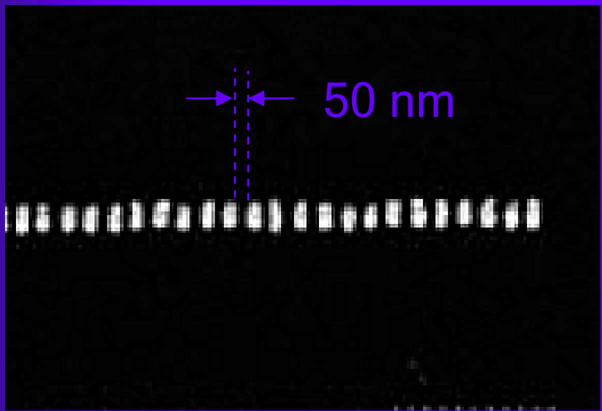
The nanoparticle waveguide is locally excited by light emanating from the tip of an near-field scanning optical microscope (NSOM).

The electromagnetic energy is transported along the waveguide towards a fluorescent dye nanosphere sitting on top of the nanoparticles.

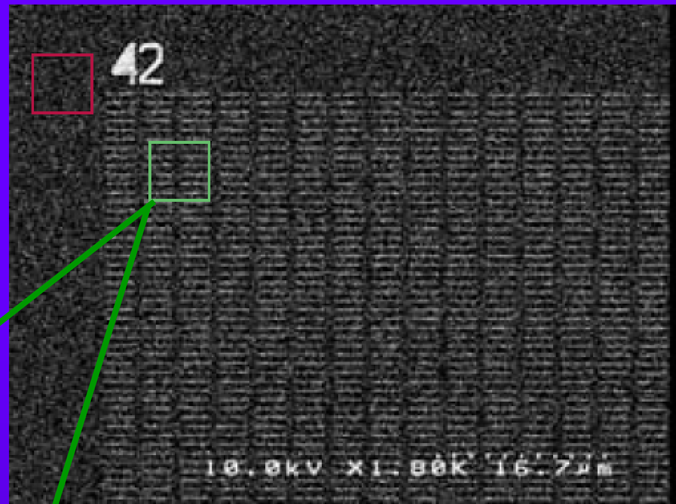


The NSOM tip is scanned along the nanoparticle chain, and the fluorescence intensity for varying tip positions along the particle chain is collected in the far-field by a photodiode.

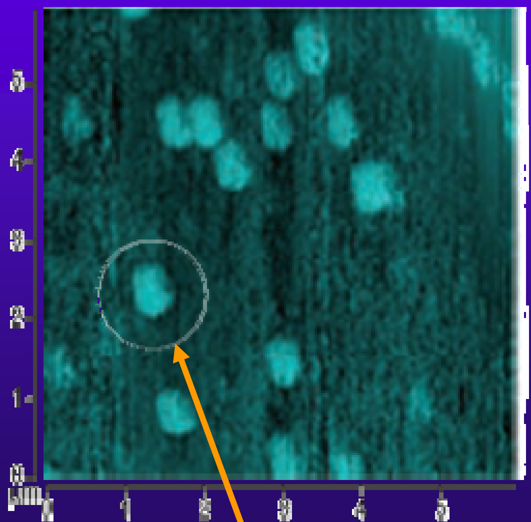
a chain of Ag anoparticles



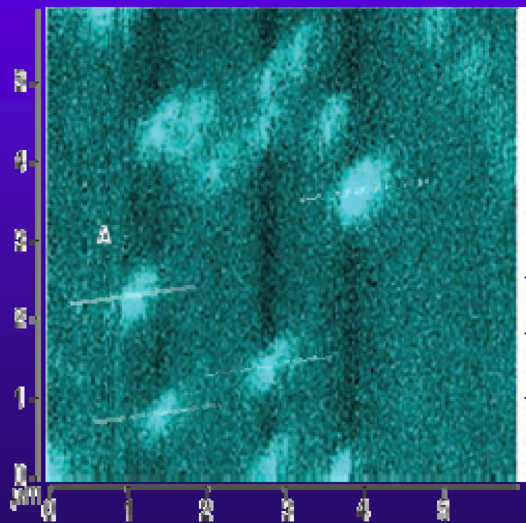
100μm × 100μm grid of plasmon waveguides



topography



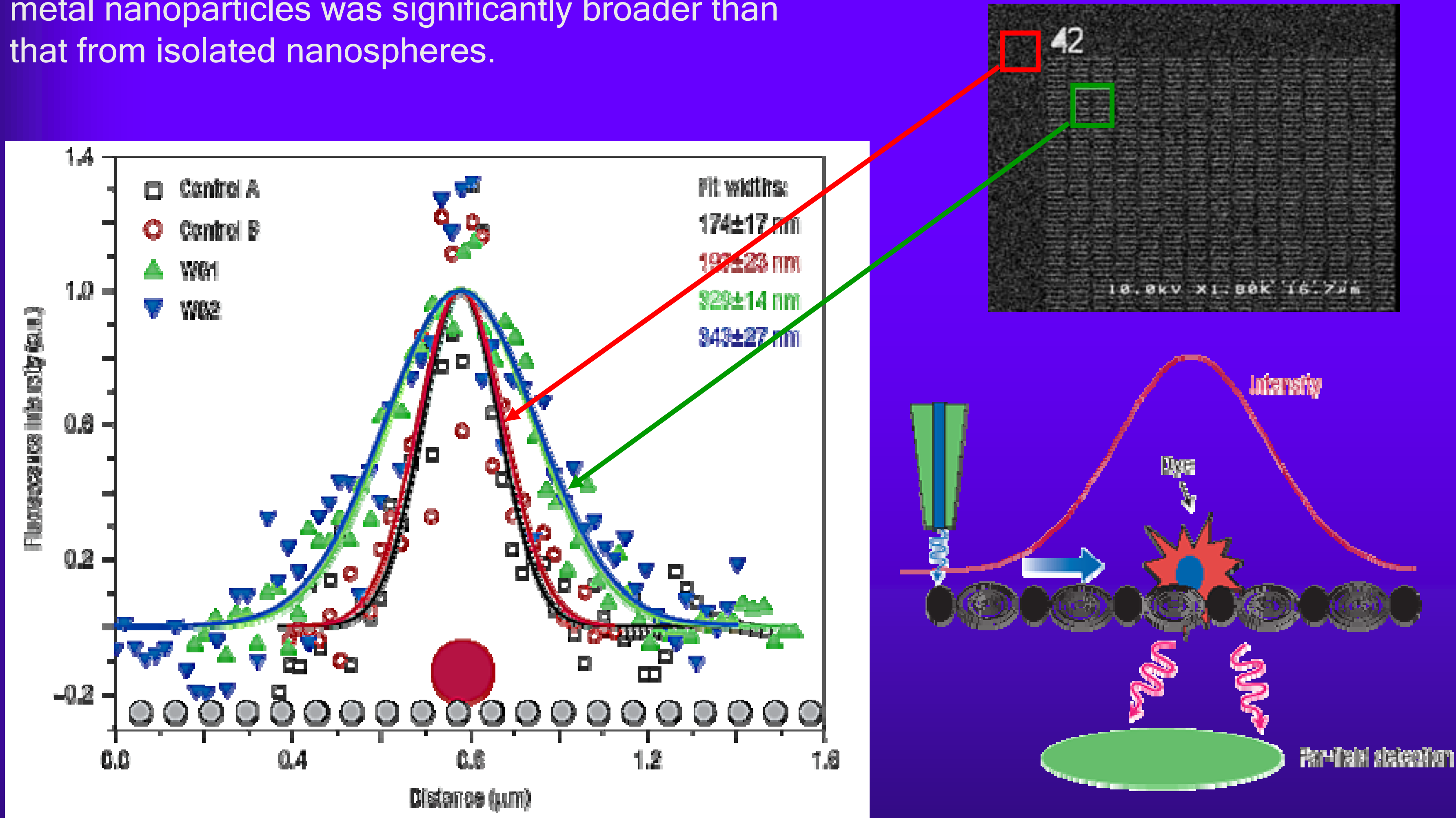
fluorecsence



fluorescent dye particles

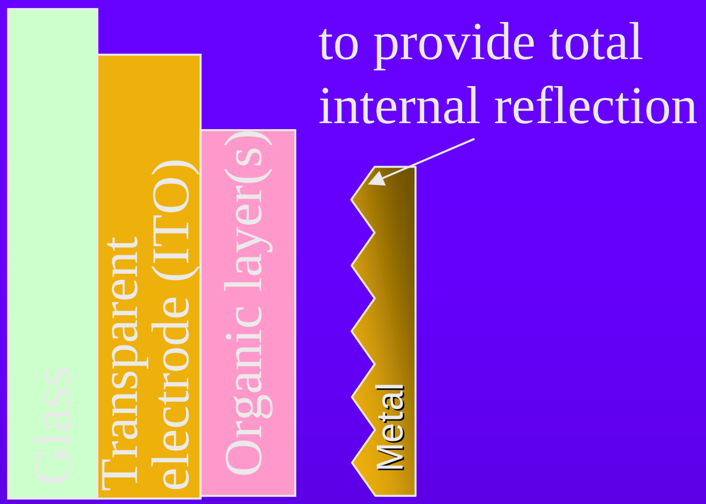
fluorescence from nanospheres sitting on top of metal nanoparticles was significantly broader than that from isolated nanospheres.

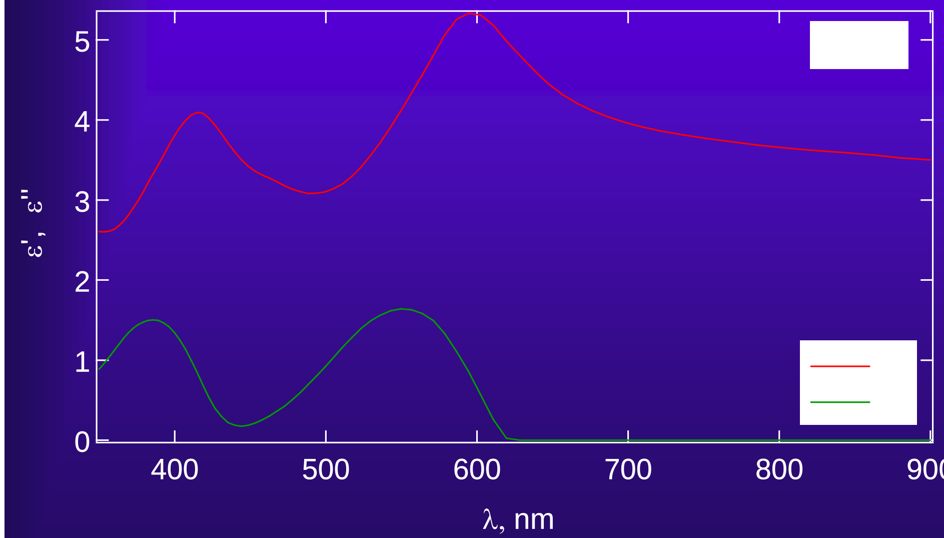
grid of plasmon waveguides

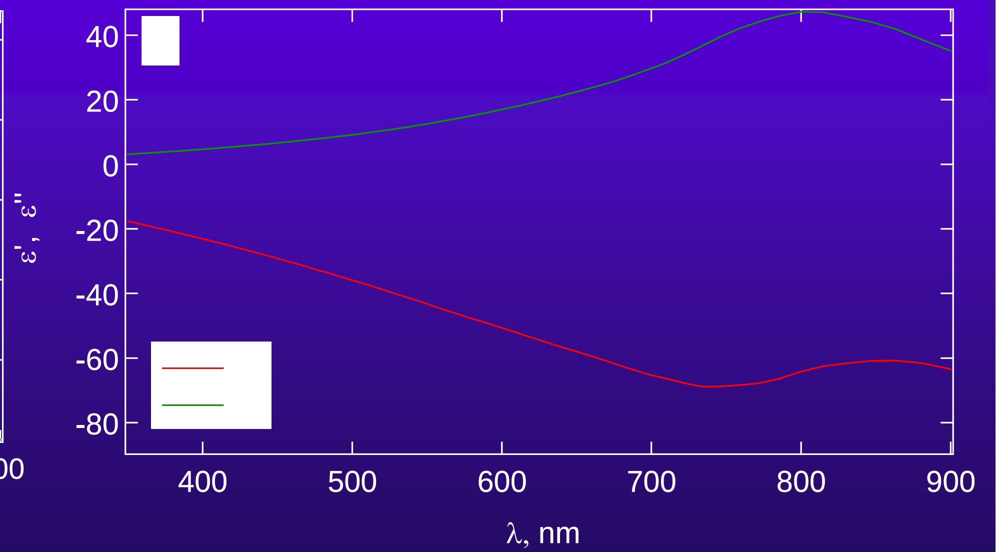


Energy transport would results in dye emission even when the microscope tip is located away from the dye, and thus manifest itself in an increased spatial width of the fluorescence spot of a dye nanosphere attached to a plasmon waveguide compared with a single free dye nanosphere.

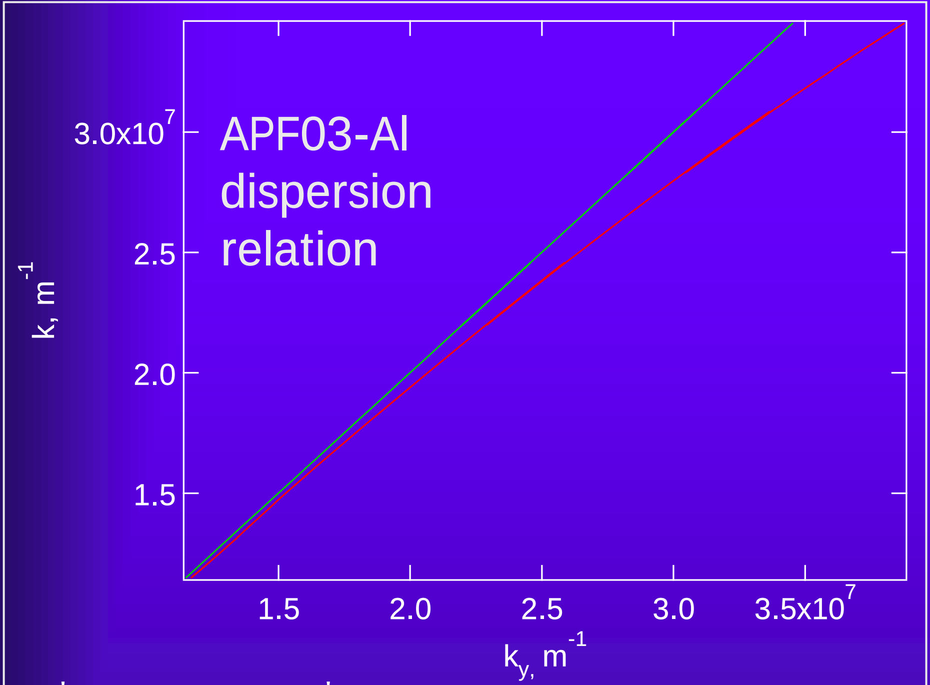
PLASMONS IN ORGANIC SOLAR CELLS







Estimation of the position of a plasmonic resonance



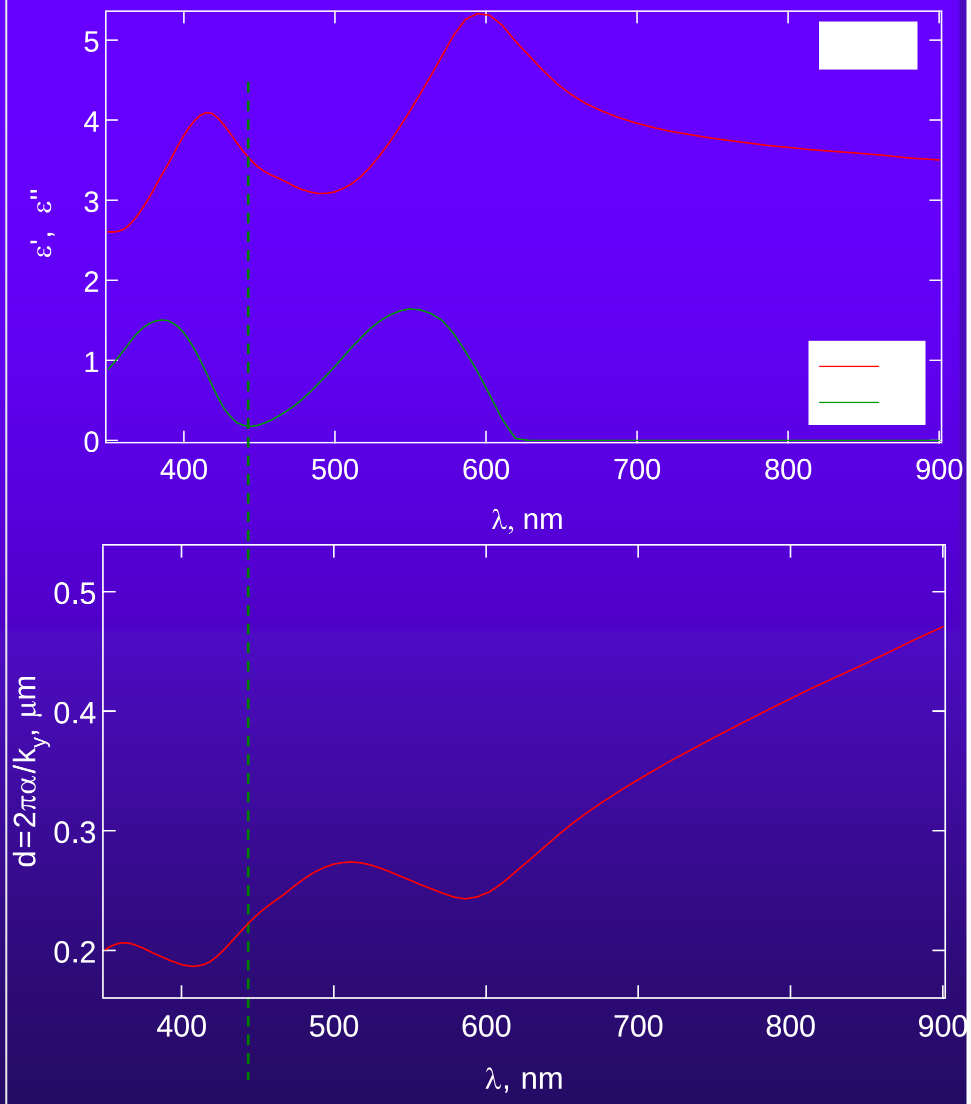
$$k'_{pl} = k_y + \Delta k'_y$$

$$\Delta k'_y = \alpha \frac{2\pi}{d}$$

normal incidence

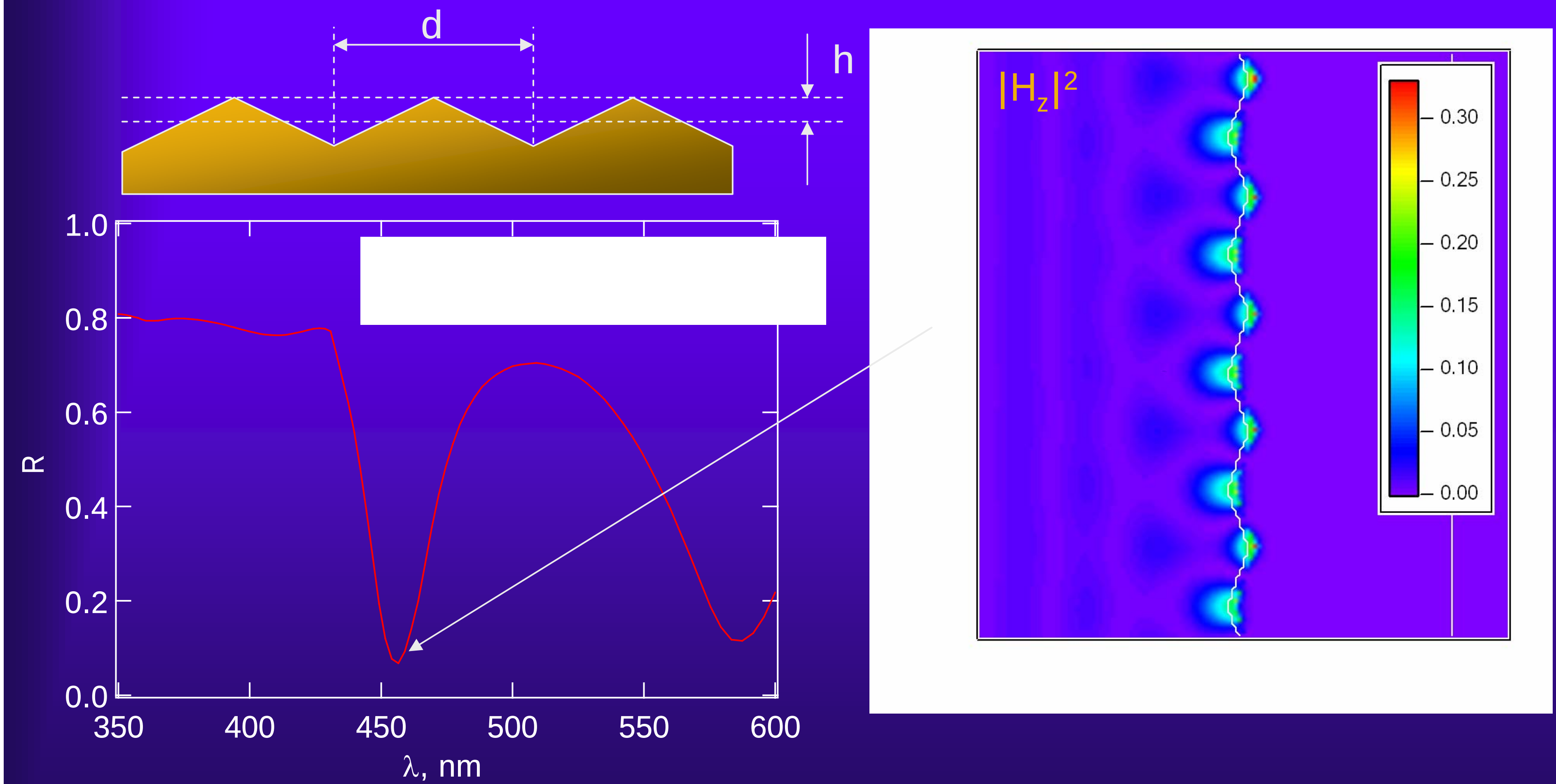
$$k_y = 0 \quad k'_{pl} = \alpha \frac{2\pi}{d}$$

where *d* is a period of grating
(sinusoidal, tiranglar or step-like)



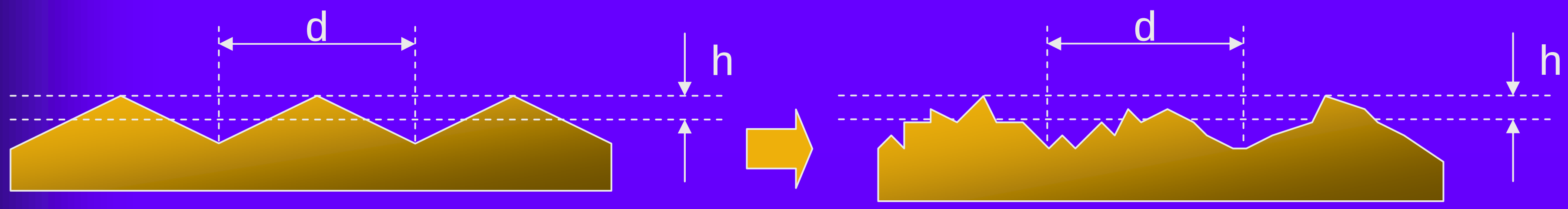
d=0.23 μm

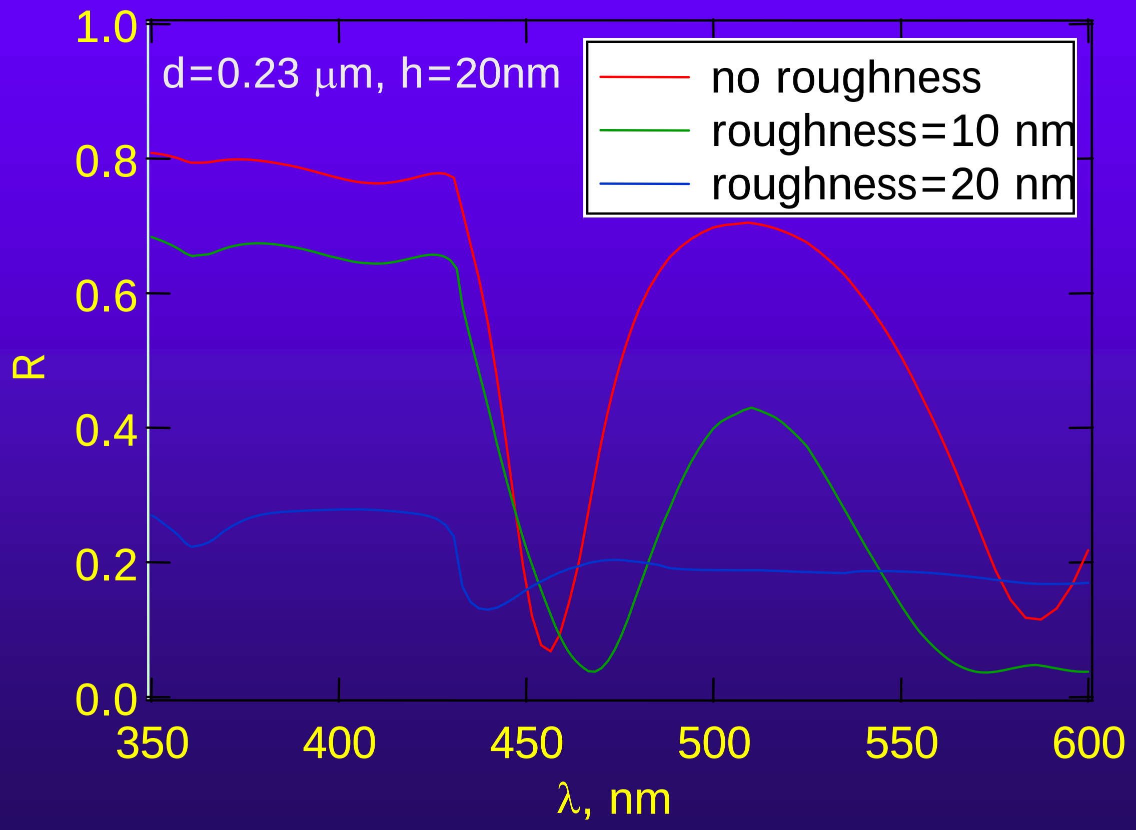
We applied the recursive Green's function technique (A.Rahachou, I.Zozoulenko, Phys. Rev. B, **72**, 155117 (2005)) to calculate spectra and intensity of the magnetic (H_z) field.



Resonance peak position agrees very well with the analytical estimation: 452 versus 450 nm

Effect of surface roughness





Full-wave simulation methods

- Method of Moments (MoM)
- Finite-Difference Time-Domain (FDTD)
- Partial-Element Equivalent-Circuit (PEEC)
- Finite Element Method (FEM)
- Transmission Line Method (TLM)

Method of Moments (MoM)

- Grid only the metal surfaces
  - grid can be non uniform
- Solve for currents on the surface cells
- Impose Maxwell Equations through the "Electric Field Integral Equation"
- Set up a linear system problem trying different combinations of the basis current functions

Finite Element Method (FEM)

- Grid entire computation volume
  - cell size small vs. minimum wavelength
  - can be non-uniform
- Solve for potentials (or fields) at the cell vertices (nodal values)
- Set up a system of equations using linear combination of trial basic functions (e.g. polinomials) for the nodal potentials
- Iterate toward mimimum energy system

Finite-Difference Time-Domain (FDTD)

- Grid all the volume of the computational domain
  - uniform grid
  - cell size small vs. minimum wavelength of interest
- Solve directly Maxwell equations for E and H using finite difference
  - solve for the fields E, H in every cell at each time step based on their values at the previous time step in that cell and in the adjacent cells.
  - implicit schemes are inefficient for partial differential equations

THANK YOU FOR VIEWING!

Maxwell's equations

The four basic equations in their differential form are as follows:

1. $\nabla \bullet D = \rho$
2. $\nabla \times H = J + (\delta D/\delta t)$
3. $\nabla \bullet B = 0$
4. $\nabla \times E = -(\delta B/\delta t)$,

where	D	=	electric displacement
	ρ	=	electric charge density
	H	=	magnetic field strength
	J	=	electric current density
	B	=	magnetic flux density
	E	=	electric field strength.