计算机科学与工程学院（网络空间安全学院）

Algorithmic Game Theory

算法博弈论

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2019 Fall

Previous

Women’s Badminton Scandal

The women’s badminton scandal at the 2012 Olympics was caused by a misalignment of the goal of the teams and that of the Olympic Committee.

Give some suggestions for how to modify the Olympic badminton tournament format to reduce or eliminate the incentive for a team to intentionally lose a match.

System Designer’s Responsibility

The burden lies on the system designer to anticipate strategic behavior, not on the participants to behave against their own interests.

Identify a real-world system in which the goals of some of the participants and the designer are fundamentally misaligned, leading to manipulative behavior by the participants.

Course Content and Calendar

Content

1. Introduction and Examples
2. Mechanism Design Basics
3. Myerson’ s Lemma
4. Algorithmic Mechanism Design
5. Selfish Routing and the Price of Anarchy
6. More on Selfish Routing

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02

Mechanism Design Basics

Lecture 2

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Introduction-Mechanism Design

Auction

An auction is a process of buying and selling goods or services by offering them up for bid, taking bids, and then selling the item to the highest bidder

Auction Theory

Auction theory is an applied branch of economics which deals with how people act in auction markets and researches the properties of auction markets

Typical Auctions

- English auction/Dutch auction
- First-price auction/Second-price auction

Single-Item Auctions

1

Course Goal

Understand how to design systems with strategic participants that have good performance guarantees

System Model

A seller with a single item for sell through an auction

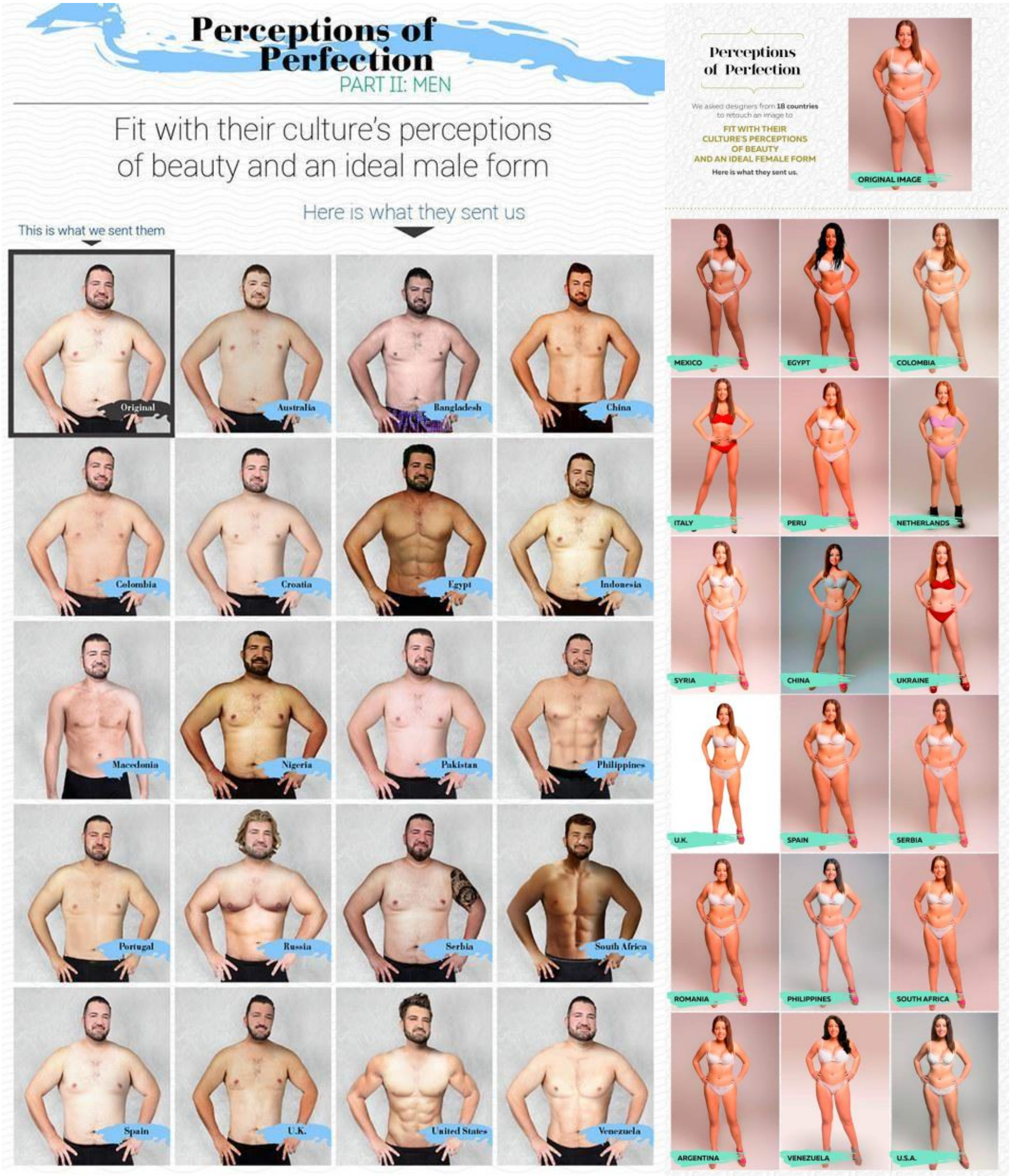
Bidders

There is n of (strategic!) bidders who are potentially interested in buying the item

Auction Formats

- Auction can be carried out by various auction formats
- We want to reason about bidder behavior in various auction formats

Key Assumptions



Independent Private Values

Valuation is private, unknown to the seller and other bidders

Nonnegative Values

Each bidder i has a nonnegative valuation v_i, the maximum willingness-to-pay for the item being sold

Bidder i wants to acquire the item as cheaply as possible, provided the selling price is at most v_i

Quasilinear Utility Model

- If a bidder i loses an auction, her utility is 0
- If a bidder wins at a price p, her utility is $v_i - p$

Auction Theory-From the Bidder’s Pont of View

How to Bid to Maximize Its Own Utility?

Auction Theory-From the Seller’ s Pont of View

Who of Them Get It?
(Who is the Winner?)

How Much Will the Winner Pay?
(To Maximize Seller’s Profit)

Sealed-Bid Auction

1) Each Bidder Submit a Bid to Maximize Utility

2) Seller Decides Who of Them Get It?
(Who is the Winner?)

3) Seller Decides How Much Will the Winner Pay?
(To Maximize Seller's Profit)

Auction Formats

There are traditionally four types of auctions that are used for the allocation of a single item

1 Open Bid Auctions

Ascending-Bid Auction

- English Auction
- Price is raised until only one bidder remains, who wins and pays the final price

Descending-bid auction

- Dutch Auction
- Price is lowered until someone accepts, who wins the object at the current price

2 Sealed Bid Auctions

First Price Auction

- Highest bidder wins; pays her bid, the highest bid

Second Price Auction

- Vickrey Auction
- Highest bidder wins; pays the second highest bid

Sealed-Bid Auction

1) Name

2) Birthday

3) Your Valuation

- (MM+DD)*0.1
- E.g. (11+08)*0.1=1.9

4) Submit Your Bids

5) Experiments

- 1st Price Bidding
- 2nd Price Bidding
- 3rd Price Bidding

Bid to Maximize Your Utility!

Second-Price Auctions and Dominant Strategies

2

Proposition 2.1 (Incentives in Second-Price Auctions)

In a second-price auction, every bidder i has a dominant strategy: set the bid b_i equal to her private valuation v_i.

Dominant Strategy

A strategy (i.e., a bid) that is guaranteed to maximize a bidder's utility, no matter what the other bidders do

Easy to Participate

When selecting a bid, a bidder doesn't need to reason about the other bidders in any way—how many there are, what their valuations are, whether or not they bid truthfully, etc

Advantage over First-Price Auction

In First-Price Auction, it never makes sense to bid one's valuation—this guarantees zero utility

The optimal amount to underbid depends on the bids of the other bidders

Proof

2 Proposition 2.1 (Incentives in Second-Price Auctions)

In a second-price auction, every bidder i has a dominant strategy: set the bid b_i equal to her private valuation v_i.

System Model

Bidder $\boldsymbol{i}$, valuation $\boldsymbol{v_i}$, bid $\boldsymbol{b_i}$, bids from the other bidders $\boldsymbol{b_{-i}}$

$$\boldsymbol{B} = \max_{i \neq -i} \boldsymbol{b_{-i}}$$

Bidder i's Utility

If lose, utility is 0; If win, utility is $\boldsymbol{v_i - p}$, where p is the second-highest bid

Case 1: $v_i < B$

Bid truthfully $\boldsymbol{b_i = v_i}$. Lose, receive utility 0

Case 2: $v_i \geq B$

Bid truthfully $\boldsymbol{b_i = v_i}$. Win, receive utility $\boldsymbol{v_i - B > 0}$

Second-Price Auctions and Dominant Strategies

2

Proposition 2.2 (Nonnegative Utility)

In a second-price auction, every truthful bidder is guaranteed nonnegative utility.

If a bidder i is the winner

Its utility is $\boldsymbol{v_i - p}$, where p is the second-highest bid. Since i is the winner (and hence the highest bidder) and bid her true valuation, $\boldsymbol{p < v_i}$ and hence $\boldsymbol{v_i - p \geq 0}$.

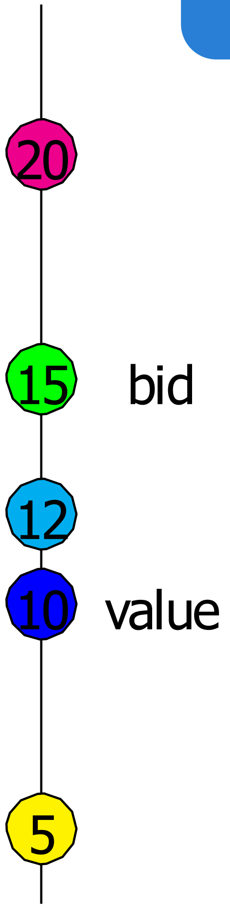
If a bidder i is the loser

Losers receive utility 0

Dominant Strategy

A truthful bidder—meaning one that bids her true valuation—never regrets participating in a second-price auction

Cheating is not the Dominant Strategy in Second Price Auctions



Bidding your value weakly dominates bidding HIGHER

Suppose your value is $10 but you bid $15. Three cases:

There is a bid higher than $15 (e.g. $20)

You loose either way: no difference

2nd highest bid is lower than $10 (e.g. $5)

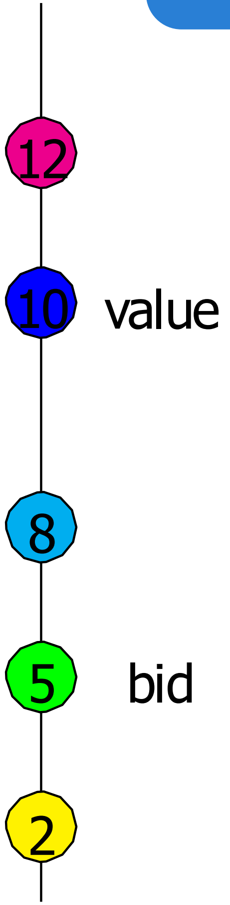
You win either way and pay $5: no difference

2nd highest bid is between $10 and $15 (e.g. $12)

- Cheating: You win with $15: lose $2
- Biding Truthfully: You loose with $10: zero payoff

Cheating is not the Dominant Strategy in Second Price Auctions

Bidding your value weakly dominates bidding LOWER



Suppose your value is $10 but you bid $5. Three cases:

There is a bid higher than $10 (e.g. $12)

You loose either way: no difference

2nd highest bid is lower than $5 (e.g. $2)

You win either way and pay $2: no difference

2nd highest bid is between $5 and $10 (e.g. $8)

- Cheating: You loose with $5: zero payoff
- Biding Truthfully: You win with $10: earn $2

Second-Price Auctions and Dominant Strategies

2

Definition 2.3 (Dominant-Strategy Incentive Compatible)

An auction is dominant-strategy incentive compatible (DSIC) if truthful bidding is always a dominant strategy for every bidder and if truthful bidders always obtain nonnegative utility

Social Welfare

The social welfare of an outcome of a single-item auction by $\sum_{i=1}^{n} v_i x_i$

Parameters

x_i is the indicator, 1 if i wins and 0 if i loses

- There is only one item: feasibility constraint $\sum_{i=1}^{n} x_i \leq 1$

Key Insights

- The social welfare is just the valuation of the winner, or 0 if there is no winner
- An auction is welfare maximizing if, when bids are truthful, the auction outcome has the maximum possible social welfare

一美元拍卖陷阱

传销不挣钱为啥还有这么多人参与？美国大选为啥总是两党之争？

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First Price Auctions

Highest bidder wins and pays her bid

Would you bid your value?

What happens if bid less than your value?

- You get a positive payoff if you win
- But your chances of winning are smaller
- Optimal bid reflects this tradeoff

Second-Price Auctions and Dominant Strategies

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Advantage over First-Price Auction

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The optimal amount to underbid depends on the bids of the other bidders

Winner’s Curse

Common Value

- Value of the item is the same for everybody
- But nobody knows the true value
- Each bidder obtains an independent and unbiased estimate of the value

How much do you bid?

- Your estimate is $1 million
- Suppose everybody, including you, bids their estimate and you are the winner

What did you just learn?

- Your estimate must have been larger than the others’
- The true value must be smaller than $1 million
- You overpaid

Winner's Curse

If everybody bids her estimate winning is "bad news": Winner's Curse

Optimal strategies are complicated

Bidders bid much less than their value to prevent winner's curse

Auction formats are not equivalent in common value auctions

Open bid auctions provide information and ameliorates winner's curse

- Bids are more aggressive

Sealed bid auctions do not provide information

- Bids are more conservative

To prevent winner's curse

Base your bid on expected value conditional on winning

Auction Design: Failures

Winning Bid	Second Highest Bid
NZ$100,000	NZ$6,000
NZ$7,000,000	NZ$5,000
NZ$1	None

Source: John McMillan, "Selling Spectrum Rights," *Journal of Economic Perspectives*, Summer 1994

New Zealand Spectrum Auction (1990)

- Used second price auction with no reserve price
- Estimated revenue NZ$ 240 million
- Actual revenue NZ$36 million

Problems

- Second price format politically problematic
- Public sees outcome as selling for less than its worth
- No reserve price

Auction Design: Failures

Initial Bid	Final Price
A$212 mil.	A$117 mil.
A$177 mil.	A$77 mil.

Source: John McMillan, "Selling Spectrum Rights," *Journal of Economic Perspectives*, Summer 1994

Australian TV License Auction (1993)

- Two satellite-TV licenses
- Used first price auction

High bidders had no intention of paying

- They bid high just to guarantee winning
- They also bid lower amounts at $5 million intervals

They defaulted

- Licenses had to be re-awarded at the next highest bid
- Those bids were also theirs

Problem

- No penalty for default

Auction Design: Failures

Turkish GSM license auction (2000)

- Two GSM 1800 licenses to be auctioned
- Round 1: First price sealed bid auction
- Round 2: First price sealed bid auction with reserve price
  - Reserve price is the winning bid of Round 1

Bids in the first round

Bidder	Bid Amount
Is-Tim	$2.525 bil.
Dogan+	$1.350 bil.
Genpa+	$1.224 bil.
Koc+	$1.207 bil.
Fiba+	$1.017 bil.

Bids in the second round

- NONE!
- All four bidders exit license auction

Problem

- Facilitates entry deterrence

Auction Design Challenge

Objective

One common objective is to maximize expected revenue

Auction Design is a Challenge When

- Values are correlated
- Bidders are risk averse
- Collusion
- Entry deterrence
- Reserve price

Auction Design Challenge

Strategies

- **Correlated values**: Ascending bid auction is better
- **Risk averse bidders**

  Second price auction: risk aversion does not matter
- **Collusion**: ?
- **Entry deterrence**: Sealed bid auctions are better to promote entry

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胆小鬼博弈

人类接近毁灭的危机：古巴导弹危机是怎么解决的?

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Ideal Auctions

2

Theorem 2.4 (Second-Price Auctions Are Ideal)

A second-price single-item auction satisfies the following

1) Strong incentive guarantees

It is a DSIC auction

2) Strong performance guarantees

It is welfare maximizing

3) Computational efficiency

It can implemented in time polynomial (indeed, linear) in the size of the input, meaning the number of bits necessary to represent the numbers $v_1, v_2 . . . , v_n$

Ideal Auctions

2

Theorem 2.4 (Second-Price Auctions Are Ideal)

A second-price single-item auction satisfies the following

1) Strong incentive guarantees

It is a DSIC auction

From Bidder's Perspective

Makes it particularly easy to choose a bid, and levels the playing field between sophisticated and unsophisticated bidders

From Bidder's Perspective

From Seller's Perspective

Makes it much easier to reason about the auction's outcome

From Seller's Perspective

Only Assumption

A bidder with an obvious dominant strategy will play it

The Only Assumption

Ideal Auctions

2

Theorem 2.4 (Second-Price Auctions Are Ideal)

A second-price single-item auction satisfies the following

Strong performance guarantees

It is welfare maximizing

DSIC is not enough

An auction that gives the item away for free to a random bidder is DSIC, but it makes no effort to identify which bidders actually want the item

DSIC is not enough

SW maximization

Even though the bidder valuations are a priori unknown to the seller, the auction nevertheless identifies the bidder with the highest valuation!

With SW maximization

Second-Price Auction

It solves the social welfare maximization problem as well as if all of the bidders' valuations were known in advance

second-price auction

Ideal Auctions

2

Theorem 2.4 (Second-Price Auctions Are Ideal)

A second-price single-item auction satisfies the following

Computational Efficiency

It can implemented in time polynomial (indeed, linear) in the size of the input, meaning the number of bits necessary to represent the numbers $v_1, v_2 . . . , v_n$

Computational Efficiency

To have potential practical utility, an auction should run in a reasonable amount of time

Computational Efficiency

Online Advertising

Auctions for online advertising, generally need to run in real time.

Online Advertising

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Background

Web Search Results

A list of organic search results

- Deemed relevant to your query by an algorithm like PageRank

A list of sponsored links, which have been paid for by advertisers

Sponsor’s Search Auction

Every time you type a search query into a search engine, an auction is run in real time to decide

- 1) Which advertisers’ links are shown
- 2) How these links are arranged visually
- 3) What the advertisers are charged

Background

Huge Revenue

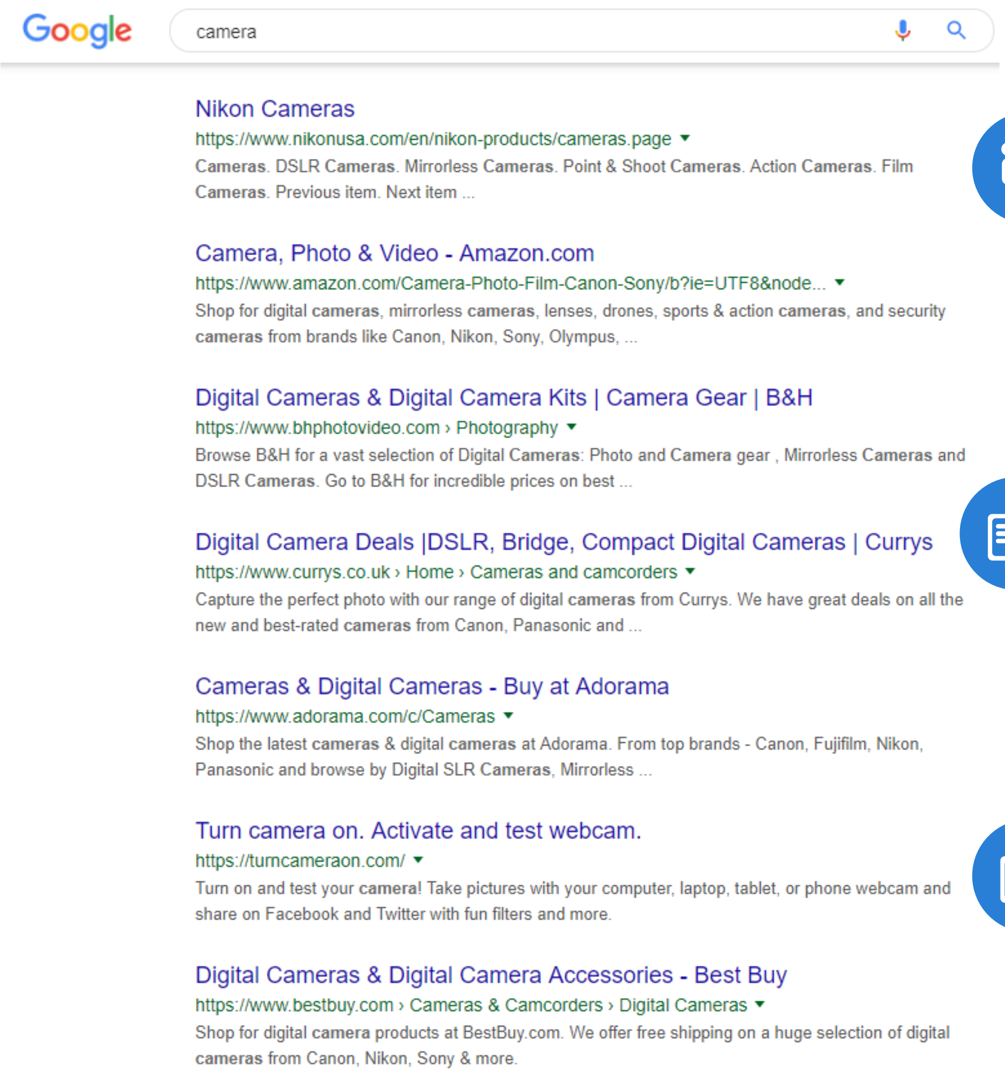
Sponsored search auctions generated roughly 98% of Google's revenue

Internet Economy

Online advertising is now sold in many different ways: social network, live streaming

Sponsored search auctions continue to generate tens of billions of dollars of revenue every year

The Basic Model of Sponsored Search Auctions



Search for One Category of Items

- The items for sale are k “slots” for sponsored links on a search results page
- The bidders are the advertisers who have a standing bid on the keyword that was searched on

Multi-Item Auction

Such auctions are more complex than single-item auctions

- First, there are generally multiple items for sale (i.e., $k > 1$)
- Second, these items are not identical (SUV, SmartPhone….)

Orders/Ranks

If ads are displayed as an ordered list, then higher slots in the list are more valuable than lower ones

- Since people generally scan the list from top to bottom

Click-Through Rates (CTRs)

Click-Through Rates (CTRs)

Quantify the difference between different slots

The CTR α_j of a slot j represents the probability that one user clicks on the link on this slot

- Or, the fraction of impression, the fraction of being shown on the page

Assumptions on Ordering of CTR

- 1) Ordering the slots from top to bottom, with the reasonable assumption that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$
- 2) Unreasonable assumption that the CTR of a slot is independent of its occupant
- 3) Bidders have private valuation v_i about the clicks on its link

Expected Value for Each Slot

- The expected value derived by advertiser i from slot j is $v_i \alpha_j$

Steps in Sponsored Search Auction

How Does the Sponsor Search Auction Carry Out?

1) There are k Slots for Auction

2) Whom Are the k Winners from the n Advertisers?

3) What Order to Put the k Winning Advertisers in the Slots?

4) How Much Should Each of the k Winning Advertisers Pay?

What We Want

Is there an ideal sponsored search auction? Our desiderata are:

1) DSIC

Truthful bidding should be a dominant strategy, and leads to non-negative utility

2) Social welfare maximization

The assignment of bidders to slots should maximize $\sum_{i=1}^{n} v_i x_i$,

- x_i denotes the CTR of the slot to which i is assigned (or 0 if i is not assigned to a slot)

Constraint: Each slot can only be assigned to one bidder, and each bidder gets only one slot

3) Computational efficiency

The running time should be polynomial (or even near-linear) in the size of the input $v_1, v_2, \dots, v_k$

Since zillions of these auctions need to be run every day!

Design Approach

2

What’s hard about auction design problems is that we have to design jointly two things:

The choice of who wins what

The choice of who pays what

Step 1

Step 2

Ensure properties (2) and (3)

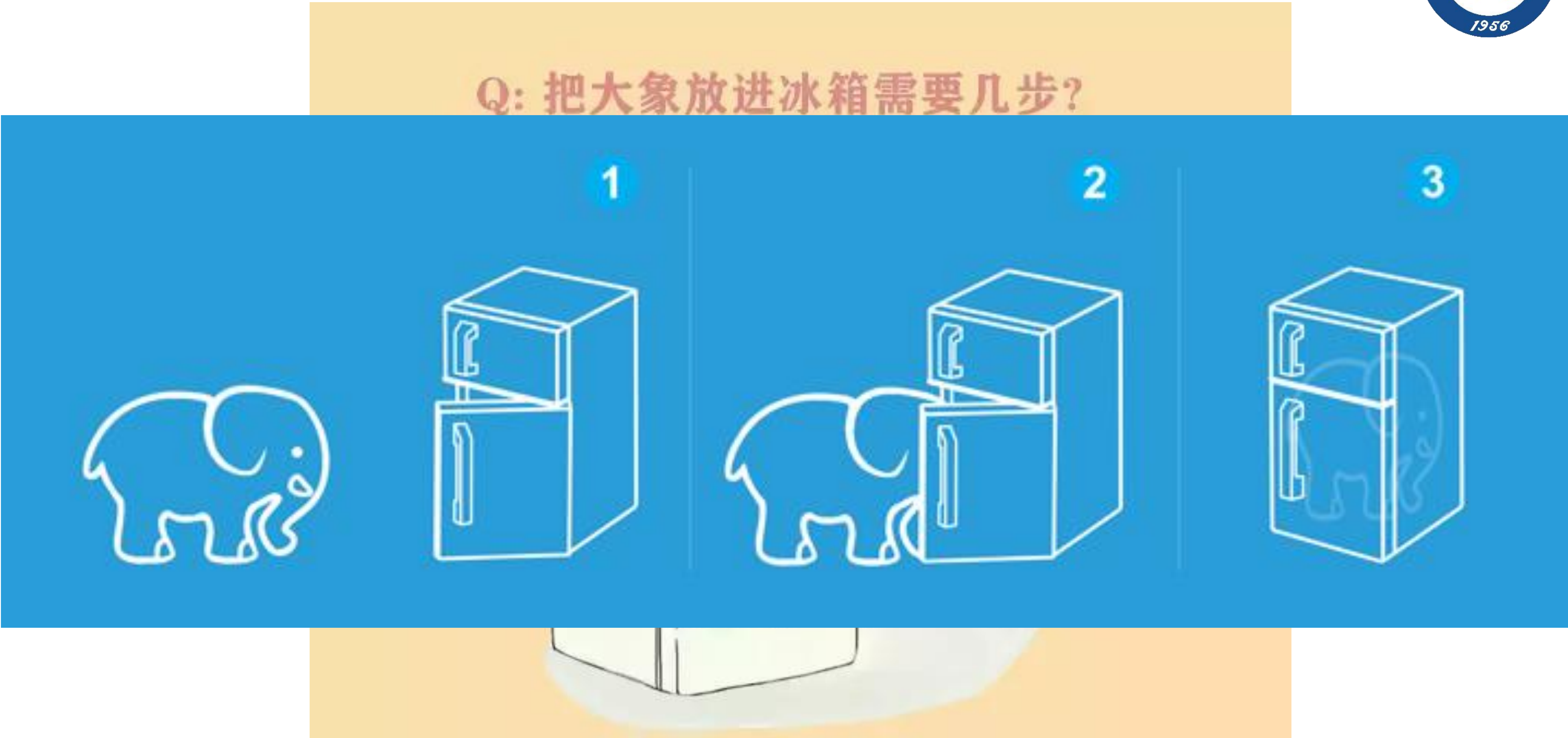
Assume, without justification, that bidders bid truthfully. Then, how should we assign bidders to slots so that the above properties (2) and (3) hold?

Ensure property (1)

Given our answer to Step 1, how should we set selling prices so that the above property (1) holds?

If we efficiently solve both of these problems, then we have constructed an ideal auction

把大象放进冰箱需要几步?



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The Upshots

Single-Item Auction

There is one seller with one item and multiple bidders with private valuations

Single-item auction design is a simple but canonical example of mechanism design

Dominant-Strategy Incentive Compatible (DSIC)

An auction is DSIC if truthful bidding is a dominant strategy and if truthful bidders always obtain nonnegative utility

Welfare Maximizing

An auction is welfare maximizing if, assuming truthful bids, the auction outcome always has the maximum possible social welfare.

The Upshots

Ideal Auction

Second-price auctions are "ideal" in that they are DSIC, welfare maximizing, and can be implemented in polynomial time

Sponsored Search Auctions

Sponsored search auctions are a huge component of the Internet economy. Such auctions are more complex than single-item auctions because there are multiple slots for sale, and these slots vary in quality

Designing Ideal Auctions

A general two-step approach to designing ideal auctions is to first assume truthful bids and understand how to allocate items to maximize the social welfare, and second to design selling prices that turn truthful bidding into a dominant strategy

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股市暴跌，为啥散户炒股票总赔钱?

看懂了这个，你再去炒股

Algorithmic Game Theory

感谢聆听

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