FURTHER QUESTIONS: FIR Filter Design

1. The ideal low-pass filter coefficients can be expressed as $h(n) = 2F_c \mathrm{sinc}(2nF_c)$ where F_c is the normalised cut-off frequency. Derive this expression from the Discrete Time Fourier Transform pair given below:

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n} d\omega \qquad \Leftrightarrow \qquad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

2. Describe the Impulse Response Truncation (IRT) method for FIR filter design and distinguish how this method differs from window-based method.

3. It is desirable to design a FIR filter that satisfies the specifications given below:

Specifications	**Value**
Passband edge frequency	1.5 kHz
Stopband attenuation	> 50 dB
Transition width	0.5 kHz
Sampling interval	8 kHz

Name of window function $w[n]$	**Transition width (Hz)**	**Ripple δ_p, δ_s**	**Mathematical definition**
Rectangular	$0.9/N$	0.089	1
Hanning	$3.1/N$	0.063	$0.5 - 0.5\cos\left[\frac{2\pi n}{N-1}\right]$
Hamming	$3.3/N$	0.0022	$0.54 - 0.46\cos\left[\frac{2\pi n}{N-1}\right]$

Table 1

Given the information in Table 1, determine the desired filter design expression and how you would realise this filter in practice.

4. In implementing a digital FIR filter, there are 4 types of noise that require considerations. Explain how the noise affects the filter outputs and approaches used to mitigate them.

5. Show that a linear phase response filter is a characteristic of the symmetrical impulse response. Explain the implications on the filter output if the phase is not linear.