Low-complexity variable loading for robust adaptive beamforming

Jie Zhuang✉, Qian Ye, Qiushi Tan and Abdulrahman Hussein Ali

Although the majority of the existing robust adaptive beamformers focus on how to choose an identical loading factor for all eigenvalues, relatively few investigations on variable loading (VL) have been conducted. A low-complexity VL beamformer is proposed in which the beamformer weight vector is deliberately prevented from converging to the noise components. Then inspired by the traditional identical loading method, the loading factor can be set in an *ad hoc* manner. Numerical results demonstrate the superior performance of the proposed beamformer relative to other existing approaches such as 'identical diagonal loading' and 'robust Capon' beamformers.

Introduction: An important topic in array signal processing is adaptive beamforming. The Capon beamformer is a representative example of conventional optimal beamformers in which the signal of interest (SOI) is allowed to pass through without distortion while the interference signals are suppressed as much as possible. However, it has been found that the Capon beamformer is subject to substantial performance degradation in the presence of modelling mismatches. This is because in such case the SOI may be treated as interference and hence be suppressed instead of being enhanced. To account for the so-called self-nulling, robust adaptive beamformers (RABs) are designed to offer acceptable array output performance. An excellent review and comparison of the existing robust techniques have been provided in [1, 2]; see also the references contained therein.

Among these RAB approaches, the diagonal loading (DL) beamformer and its extension versions may be the most common. The DL beamformers aim to utilise a loading factor (often an identical loading factor for all eigenvalues) so as to *detune* the beamformer response within the mainlobe. However, the main adverse side-effect associated with the identical loading approaches is the loss in adaptive interference suppression and noise reduction. To obtain a better balance between the robustness and adaptivity, a variable loading (VL) method is presented in this Letter. First, the beamformer weight vector is deliberately prevented from converging to the noise components, implying that the robustness can be guaranteed. Then inspired by the traditional identical loading method, the loading factor is set in an *ad hoc* manner and hence the proposed method is of low-complexity.

Problem formulation: The well-known Capon beamformer can be formulated by the following optimisation problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}\boldsymbol{w} \text{ s.t. } \boldsymbol{w}^{\mathrm{H}}\bar{\boldsymbol{a}} = 1 \tag{1}$$

with the solution (after omitting the immaterial scaling factor)

$$\boldsymbol{w}_{\mathrm{c}} = \widehat{\boldsymbol{R}}^{-1}\bar{\boldsymbol{a}} \tag{2}$$

where $\bar{\boldsymbol{a}}$ denotes the nominal SOI steering vector and $(\cdot)^{\mathrm{H}}$ represents the Hermitian transpose. The matrix $\widehat{\boldsymbol{R}}$ represents the data covariance matrix estimated by $\widehat{\boldsymbol{R}} = (1/K)\sum_{k=1}^{K}\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{H}}(k)$ where $\{\boldsymbol{x}(k)\}_{k=1}^{K}$ denote the array observations or snapshots. In the presence of mismatch between the true SOI steering vector and its nominal version, however, the SOI may be treated as an interference signal and consequently be suppressed instead of being enhanced, leading to 'self-cancellation'.

To penalise the imperfections of the data covariance matrix estimate due to small snapshot number as well as imperfections in the knowledge of the SOI steering vector [1], a regularisation term is added to the objective function of (1), formulating the DL beamformer

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}\boldsymbol{w} + \gamma\|\boldsymbol{w}\|^2 \text{ s.t. } \boldsymbol{w}^{\mathrm{H}}\bar{\boldsymbol{a}} = 1 \tag{3}$$

with the solution given by

$$\boldsymbol{w}_{\mathrm{DL}} = (\widehat{\boldsymbol{R}} + \gamma\boldsymbol{I})^{-1}\bar{\boldsymbol{a}}. \tag{4}$$

The loading factor γ can be chosen in an *ad hoc* way, typically $\gamma = 10\sigma_n^2$ where σ_n^2 denotes the noise power.

The worst-case optimisation-based beamformer [3] is a popular RAB which intends to minimise the output power while forcing the magnitude response for the SOI (whose steering vector lies in a known uncertainty set) to exceed unity. The optimisation problem presented in [3] is as follows:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}\boldsymbol{w} \quad \text{s.t.} \min_{\boldsymbol{c}\in\mathcal{A}(\epsilon)} |\boldsymbol{w}^{\mathrm{H}}\boldsymbol{c}| \geq 1 \tag{5}$$

where $\mathcal{A}(\epsilon) = \{\boldsymbol{c}|\boldsymbol{c} = \bar{\boldsymbol{a}} + \boldsymbol{e},\ \|\boldsymbol{e}\| \leq \epsilon\}$ denotes the spherical uncertainty set. Interestingly, it has been shown in [4] that the RAB (5) is also a DL-type beamformer in which the loading factor γ can be calculated by solving the equation $\|\left(\boldsymbol{I} + (1/\gamma)\widehat{\boldsymbol{R}}\right)^{-1}\bar{\boldsymbol{a}}\| = \epsilon$. In addition, the simulation results in [3] have shown that the traditional DL beamformer (where $\gamma = 10\sigma_n^2$) performs as well as the RAB of [3].

Performing eigendecomposition on $\widehat{\boldsymbol{R}}$ yields

$$\widehat{\boldsymbol{R}} = \widehat{\boldsymbol{U}}\widehat{\boldsymbol{\Lambda}}\widehat{\boldsymbol{U}}^{\mathrm{H}} = \sum_{i=1}^{N}\widehat{\lambda}_i\widehat{\boldsymbol{u}}_i\widehat{\boldsymbol{u}}_i^{\mathrm{H}} \tag{6}$$

where N is the array sensor number. The matrix $\widehat{\boldsymbol{U}} = \left[\widehat{\boldsymbol{u}}_1 \quad \cdots \quad \widehat{\boldsymbol{u}}_N\right]$ collects all the eigenvectors, and $\widehat{\boldsymbol{\Lambda}} = \mathrm{diag}\{\widehat{\lambda}_1,\ \ldots,\ \widehat{\lambda}_N\}$ is a diagonal matrix with the eigenvalues $\widehat{\lambda}_1 \geq \ldots \geq \widehat{\lambda}_N$ being non-increasingly ordered. Ideally, the subdominant eigenvalues corresponding to the noise subspace should be equal to σ_n^2. Now, the DL weight vector $\boldsymbol{w}_{\mathrm{DL}}$ in (4) can be rewritten as

$$\boldsymbol{w}_{\mathrm{DL}} = \sum_{i=1}^{N}\frac{(\widehat{\boldsymbol{u}}_i^{\mathrm{H}}\bar{\boldsymbol{a}})}{\widehat{\lambda}_i + \gamma}\widehat{\boldsymbol{u}}_i \tag{7}$$

From the above, it can be seen that for large eigenvalues the term $(\widehat{\boldsymbol{u}}_i^{\mathrm{H}}\bar{\boldsymbol{a}})/(\widehat{\lambda}_i + \gamma)$ is almost unchanged whether γ is loaded or not. However, for small eigenvalues the term $(\widehat{\boldsymbol{u}}_i^{\mathrm{H}}\bar{\boldsymbol{a}})/(\widehat{\lambda}_i + \gamma)$ reduces significantly once γ is loaded. This implies that the effect of the loading factor γ is to de-emphasise components corresponding to small eigenvalues (i.e. the noise components). By doing so, the DL weight vector contains less components orthogonal with the SOI (since the SOI is located in the signal subspace), thereby avoiding the self-nulling. However, this robustness is gained at the cost of adaptive interference suppression and noise reduction.

Proposed VL beamformer: To make a better trade-off between the robustness and adaptivity, the optimisation problem considered in this Letter takes the following form:

$$\min_{\boldsymbol{w}} \boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}\boldsymbol{w} + \gamma\boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}^{-1}\boldsymbol{w} \text{ s.t. } \boldsymbol{w}^{\mathrm{H}}\bar{\boldsymbol{a}} = 1 \tag{8}$$

with the solution given by

$$\boldsymbol{w}_{\mathrm{VL}} = (\widehat{\boldsymbol{R}} + \gamma\widehat{\boldsymbol{R}}^{-1})^{-1}\bar{\boldsymbol{a}}. \tag{9}$$

In comparison with (3), the term $\gamma\boldsymbol{w}^{\mathrm{H}}\widehat{\boldsymbol{R}}^{-1}\boldsymbol{w}$ in (8) can be viewed as a weighting function used to deliberately prevent the weight vector from converging to the noise components.

In [5], it is suggested to use the RAB method in [4] to compute a loading factor denoted by γ_{rab}. Then the loading factor for (9) is set as $\gamma = \gamma_{\mathrm{rab}}^2$. Building on the work of [5], two improvements are provided in this Letter. First, the weight vector in (9) can be rewritten as

$$\boldsymbol{w}_{\mathrm{VL}} = \sum_{i=1}^{N}\frac{(\widehat{\boldsymbol{u}}_i^{\mathrm{H}}\bar{\boldsymbol{a}})}{\widehat{\lambda}_i + (\gamma/\widehat{\lambda}_i)}\widehat{\boldsymbol{u}}_i. \tag{10}$$

Bearing in mind that the *ad hoc* DL beamformer uses the loading factor $10\sigma_n^2$ to de-emphasise the noise components and it can achieve almost the same performance as the RABs in [3, 4], the loading factor in (10) can be set as $\gamma = 10\sigma_n^4$ such that the final loading factor $\gamma/\widehat{\lambda}_i$ for the subdominant eigenvalues is also $10\sigma_n^2$. For the dominant eigenvalues, the final loading factor $\gamma/\widehat{\lambda}_i$ is much $<10\sigma_n^2$, which implies that the VL in (10) has better ability of adaptive interference suppression than that in (7). Second, in order to remove the effect of the noise perturbation, a priori knowledge of the noise power is used as the eigenvalue threshold. That is the eigenvalues used in (10) are actually $\widehat{\lambda}_i = \max\{\widehat{\lambda}_i,\ \sigma_n^2\},\ i = 1,\ \ldots,\ N$.

To summarise, the proposed low-complexity VL beamformer consists the following steps:

(i) Estimate the data covariance matrix by using the collected snapshots and then perform eigendecomposition to obtain all the eigenvectors and eigenvalues.
(ii) Replace all the eigenvalues by $\widehat{\lambda}_i = \max\{\widehat{\lambda}_i,\ \sigma_n^2\},\ i = 1,\ \ldots,\ N$.
(iii) Compute the weight vector using (10) with $\gamma = 10\sigma_n^4$.

Simulation results: Assume that one SOI and two interferers are incident on a uniform linear array with $N = 10$ isotropic sensors and half-wavelength sensor spacing. The two interference signals are from $[30°,\ 50°]$ with the input interference-to-noise ratios (INRs) in a single sensor 30 dB. The input signal-to-noise ratio is -10 dB and the nominal direction-of-arrival (DOA) of the SOI is 0°. Four other robust methods are compared with the proposed approach in terms of the array output signal-to-interference-plus-noise ratio (SINR): (i) the traditional DL with $\gamma = 10\sigma_n^2$; (ii) the RAB proposed in [4] where the uncertainty level ε is equal to $0.3N$; (iii) a recent RAB presented in [2] where the assumed SOI angular range is $[-5°, 5°]$; and (iv) the traditional VL proposed in [5] where the loading factor $\gamma = \gamma_{\text{rab}}^2$ with γ_{rab} computed by the RAB of [4]. Note that in [4] it has been proven that the two RABs proposed in [3, 4] are equivalent and the parameter $\varepsilon = 0.3N$ is suggested in [3]. For each scenario, the average of 200 independent runs is used to plot each simulation point. For reference, the optimal SINR is also plotted.

In the first example, the case of look direction error is considered. In Fig. 1, the actual SOI DOA varies from $-5°$ to 5° (i.e. the look direction error changes from $-5°$ to $+5°$) and the available snapshot number is $K = 50$. In Fig. 2, the actual SOI DOA is fixed at 3° (i.e. 3° look direction error) and the snapshot number varies from 10 to 100.

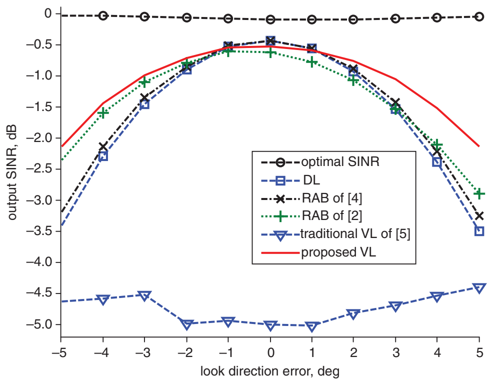

Fig. 1 *Output SINR against look direction error; first example*

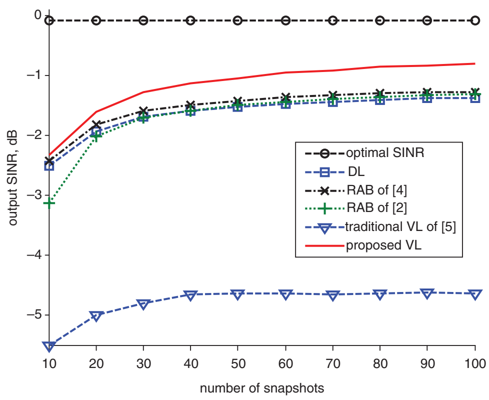

Fig. 2 *Output SINR against snapshot number; first example*

In the second example, the finite snapshot effect is also tested as depicted in Fig. 3. Here the actual SOI steering vector is formed by five coherent signal paths as $\boldsymbol{a} = \boldsymbol{a}(\theta_0) + \sum_{i=1}^{4} e^{j\phi_i}\boldsymbol{a}_i(\widetilde{\theta}_i)$ where $\theta_0 = 0°$ is the DOA of the direct path, whereas $\widetilde{\theta}_i$ corresponds to the ith coherently scattered path. The parameters $\{\phi_i\}$ represent the path phases that are independently and uniformly drawn from the interval $[0,\ 2\pi]$ in each simulation run. The angles $\{\widetilde{\theta}_i\}$ are independently drawn in each simulation run uniformly from the interval $[-5°,\ 5°]$. Note that $\{\widetilde{\theta}_i\}$ and $\{\phi_i\}$ vary from run to run while keeping unchanged from snapshot to snapshot.

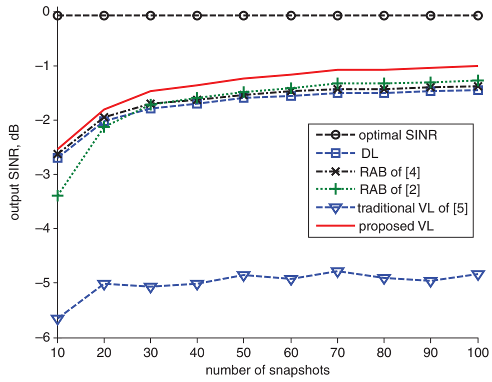

Fig. 3 *Output SINR against snapshot number; second example*

As illustrated in Figs. 1–3, the proposed VL beamformer consistently enjoys the best performance among the RAB methods tested. Note that such performance is achieved with low computational complexity.

Conclusion: A simple but effective VL beamformer is proposed in which the a priori information required is the noise power only. The proposed method can obtain the same robustness as the tradition DL methods but better flexibility in adaptive interference cancellation.

Acknowledgments: This work has been supported in part by the National Natural Science Foundation of China (NSFC) under grant no. 61571090 and the Fundamental Research Funds for the Central Universities under grant no. ZYGX2014J007.

Submitted: *4 November 2015* E-first: *5 February 2016*
doi: 10.1049/el.2015.3844
One or more of the Figures in this Letter are available in colour online.

Jie Zhuang, Qian Ye, Qiushi Tan and Abdulrahman Hussein Ali (*School of Communication and Information Engineering, University of Electronic Science and Technology of China (UESTC), Chengdu 611731, People's Republic of China*)

✉ E-mail: jz@uestc.edu.cn

References

1 Vorobyov, S.A.: 'Principles of minimum variance robust adaptive beamforming design', *Signal Process.*, 2013, **93**, (12), pp. 3264–3277
2 Khabbazibasmenj, A., Vorobyov, S.A., and Hassanien, A.: 'Robust adaptive beamforming based on steering vector estimation with as little as possible prior information', *IEEE Trans. Signal Process.*, 2012, **60**, (6), pp. 2974–2987
3 Vorobyov, S.A., Gershman, A.B., and Luo, Z.-Q.: 'Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem', *IEEE Trans. Signal Process.*, 2003, **51**, (7), pp. 313–324
4 Li, J., Stoica, P., and Wang, Z.: 'On robust Capon beamforming and diagonal loading', *IEEE Trans. Signal Process.*, 2003, **51**, (2), pp. 1702–1715
5 Gu, J., and Wolfe, P.J.: 'Robust adaptive beamforming using variable loading'. IEEE Workshop on Sensor Array and Multichannel Processing, Waltham, MA, USA, July 2006, pp. 1–5