Extension of the Signal-Subspace Projection Method to Multi-dimension Using CCA

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***Abstract*—The traditional signal-subspace projection (SSP) method combats the problem of array manifold uncertainty to gain the robustness by means of projecting the nominal manifold vector onto the signal subspace so as to eliminate the errors lying in the noise subspace. The main contribution of this paper is to extent the SSP approach from one dimension to multi-dimension. We assume that the actual manifold vector of the desired signal can be expressed as a product of a known matrix and an unknown coordinate vector. Then it is shown that the SSP method can be derived from the perspective of a problem of canonical correlation analysis (CCA) where the dimension of one subspace is one. When the dimension of the subspace (which the actual manifold of the desired signal belongs to) increases to multi-dimension, a novel projection method is developed, which can be viewed as the extension of the SSP method from one dimension to multi-dimension. Numerical results demonstrate the superiority of the proposed beamformer relatively to the conventional SSP method.**

***Keywords*—Array processing; signal-subspace projection (SSP) Method; canonical correlation analysis (CCA); robust array beamforming**

I. Introduction

Consider an array of N sensors collecting $M+1$ uncorrelated narrowband signals (one desired signal and M interference signals) located in the far-field. The second order statistics of the received $N \times 1$ signal-vector $\mathbf{x}(t)$ is represented by the covariance matrix $\mathbf{R}$

$$\mathbf{R} = \mathcal{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \sigma_d^2\mathbf{a}_d\mathbf{a}_d^H + \underbrace{\sum_{i=1}^{M}\sigma_i^2\mathbf{a}_i\mathbf{a}_i^{\mathrm{H}} + \sigma_n^2\mathbf{I}_N}_{\triangleq\mathbf{R}_{i+n}} \quad (1)$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator, σ_d^2 and $\{\sigma_i^2, i = 1, \ldots, M\}$ are the powers of the desired signal and the interferences, and $\mathbf{I}_N$ represents the $N \times N$ identity matrix. $\mathbf{a}_d$ and $\{\mathbf{a}_i, i = 1, \ldots, M\}$ stand for the manifold vectors (or steering vectors) of the desired signal and the M interferences respectively, which are the array response at the direction-of-arrival (DOA) of θ_d and $\{\theta_i, i = 1, \ldots, M\}$. The matrix $\mathbf{R}_{i+n}$ is referred to as the desired-signal-absent covariance matrix in which the effects of the desired signal are totally removed. In practical applications, the theoretical covariance matrix is normally unavailable and we have to use its estimated version instead, which may be obtained as follows

$$\widehat{\mathbf{R}} = \frac{1}{K}\sum_{l=1}^{K}\mathbf{x}(t_k)\mathbf{x}^H(t_k) \quad (2)$$

where $\{\mathbf{x}(t_k), k = 1, \ldots, K\}$ denote the K received snapshots. Performing eigen-decomposition on $\mathbf{R}$ yields

$$\begin{aligned}\mathbf{R} &= \sum_{i=1}^{N}\gamma_i\mathbf{e}_i\mathbf{e}_i^H \\ &= \mathbf{E}_s\mathbf{D}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{D}_n\mathbf{E}_n^H \end{aligned}\quad (3)$$

where the eigenvalues $\{\gamma_i, i = 1, \ldots, N\}$ are arranged in decreasing order (i.e., $\gamma_1 \geq \ldots \geq \gamma_N$), $\mathbf{e}_i$ is the eigenvector associated with γ_i. $\mathbf{R}$ can be split into two parts where $\mathbf{D}_s = \mathrm{diag}\{\gamma_1, \cdots, \gamma_{M+1}\}$ and $\mathbf{D}_n = \mathrm{diag}\{\gamma_{M+2}, \cdots, \gamma_N\}$ are diagonal matrices, $\mathbf{E}_s$ and $\mathbf{E}_n$ contain, respectively, the $M+1$ dominant eigenvectors and the remaining eigenvectors, i.e.,

$$\begin{aligned}\mathbf{E}_s &= \begin{bmatrix}\mathbf{e}_1 & \cdots & \mathbf{e}_{M+1}\end{bmatrix} \\ \mathbf{E}_n &= \begin{bmatrix}\mathbf{e}_{M+2} & \cdots & \mathbf{e}_N\end{bmatrix}\end{aligned} \quad (4)$$

Commonly $\mathbf{E}_s$ and $\mathbf{E}_n$ are referred to as the signal-subspace eigenvectors and noise-subspace eigenvectors.

An important topic in array signal processing is concerned with maximizing the signal-to-interference-plus-noise ratio (SINR), which can be expressed as the following optimization problem

$$\mathrm{SINR}_{\mathrm{max}} = \max_{\mathbf{w}}\frac{\sigma_d^2\mathbf{w}^H\mathbf{a}_d\mathbf{a}_d^H\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}} \quad (5)$$

The optimal solution is easily derived:

$$\mathbf{w}_{\mathrm{opt}} = \beta\mathbf{R}_{i+n}^{-1}\mathbf{a}_d \quad (6)$$

where the scalar constant β does not affect the array output SINR. In practical applications, the received signals often consist of the desired signal and the interferences simultaneously, which means that the desired-signal-absent covariance matrix $\mathbf{R}_{i+n}$ is unavailable or difficult to find. Therefore the desired-signal-present covariance matrix $\mathbf{R}$ is used instead of $\mathbf{R}_{i+n}$, i.e.,

$$\mathbf{w}_{\mathrm{Capon}} = \beta\mathbf{R}^{-1}\mathbf{a}_d \quad (7)$$

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which is the solution of the well known Capon beamformer or the minimum variance distortionless response (MVDR) beamformer when β is chosen such that the array gain in the desired direction θ_d is unity (i.e., $\mid \mathbf{w}_{\text{Capon}}^H \mathbf{a}_d \mid = 1 \Rightarrow \beta = (\mathbf{a}_d^H \mathbf{R}^{-1} \mathbf{a}_d)^{-1}$). Substituting (1) into (7) and using the matrix inversion lemma [1], it is found

$$\mathbf{w}_{\text{Capon}} = \frac{\beta}{1 + \sigma_d^2 \mathbf{a}_d^H \mathbf{R}_{i+n}^{-1} \mathbf{a}_d} \mathbf{R}_{i+n}^{-1} \mathbf{a}_d \tag{8}$$

The above equation shows that if the perfect $\mathbf{R}$ and $\mathbf{a}_d$ are used, the weight vectors in (6) and (7) differ by a scalar factor only and therefore the maximum SINR can also be achieved by using (7). However, the weight vector actually used by the beamformers is given by

$$\widehat{\mathbf{w}}_{\text{Capon}} = \beta \widehat{\mathbf{R}}^{-1} \mathbf{a}_0 \tag{9}$$

where the vector $\mathbf{a}_0$ denotes the presumed manifold vector corresponding to the nominal DOA θ_0. The SINR performance of the Capon beamformers may suffer a substantial degradation in the case when there exist mismatch between the presumed $\mathbf{a}_0$ (or $\widehat{\mathbf{R}}$) and its actual value $\mathbf{a}_d$ (or $\mathbf{R}$) [2]. It has been shown in [3] that the average of the output SINR loss due to the difference between the covariance matrices $\widehat{\mathbf{R}}$ and $\mathbf{R}$ is analogous to the SINR loss caused by the mismatch between $\mathbf{a}_0$ and $\mathbf{a}_d$.

To mitigate the array performance degradation caused by the difference between the actual manifold $\mathbf{a}_d$ and its nominal version $\mathbf{a}_0$, a popular robust beamformer called the signal-subspace projection (SSP) method has been proposed in [3], in which the presumed manifold $\mathbf{a}_0$ is projected onto the signal subspace so as to eliminate the mismatch lying in the noise subspace. The weight vector of the SSP can be written as

$$\widehat{\mathbf{w}}_{\text{SSP}} = \beta \widehat{\mathbf{R}}^{-1} \mathbf{P}_{\widehat{\mathbf{E}}_s} \mathbf{a}_0 \tag{10}$$

where $\mathbf{P}_{\widehat{\mathbf{E}}_s} = \widehat{\mathbf{E}}_s \widehat{\mathbf{E}}_s^H$ is the orthogonal signal-subspace projection matrix. What follows is our proposed method in which the traditional SSP method is extended from one dimension to multi-dimension by using the technique of canonical correlation analysis (CCA).

II. Proposed Method Using Canonical Correlation Analysis (CCA)

A. Steering Vector Uncertainty

Let us consider the fact that in many cases the true manifold vector $\mathbf{a}_d$ can be assumed to belong to a p-dimension linear subspace $\langle \mathbf{H} \rangle$ spanned by a known matrix $\mathbf{H} \in \mathcal{C}^{N \times p}$ with full column rank (where $p < N$). However, how to combine $\mathbf{a}_d$ using the base of $\langle \mathbf{H} \rangle$ is otherwise unknown [4], [5]. That is

$$\mathbf{a}_d = \mathbf{H}\mathbf{b} \tag{11}$$

where $\mathbf{H}$ is known, but the coordinate vector $\mathbf{b} \in \mathcal{C}^{p \times 1}$ is unknown to the beamformers. The most obvious example may be the case of the look direction error. Using the Taylor expansion and retaining the terms up to the second order, the actual manifold can be approximated by

$$\mathbf{a}(\theta_d) \approx \mathbf{a}(\theta_0) + (\theta_d - \theta_0)\dot{\mathbf{a}}(\theta_0) + \frac{(\theta_d - \theta_0)^2}{2} \ddot{\mathbf{a}}(\theta_0) \tag{12}$$

where $\dot{\mathbf{a}}(\theta_0)$ and $\ddot{\mathbf{a}}(\theta_0)$ denote the first and second derivatives with respect to the nominal DOA θ_0. Thus we can readily derive a subspace matrix $\mathbf{H}_1 = \begin{bmatrix} \mathbf{a}(\theta_0) & \dot{\mathbf{a}}(\theta_0) & \ddot{\mathbf{a}}(\theta_0) \end{bmatrix}$ such that $\mathbf{a}_0 \in \langle \mathbf{H}_1 \rangle$. The example of local scattering can be found in [5] and more examples are discussed in [4].

Also, in [6], [7] the flat ellipsoidal uncertainty set has been investigated, in which the true manifold vector is expressed as

$$\mathbf{a}_d = \mathbf{a}_0 + \mathbf{B}\mathbf{b}_{\text{fe}} = \begin{bmatrix} \mathbf{a}_0 & \mathbf{B} \end{bmatrix} \begin{bmatrix} 1 & \mathbf{b}_{\text{fe}}^T \end{bmatrix}^T, \quad \|\mathbf{b}_{\text{fe}}\| \leq 1 \tag{13}$$

where the known $N \times (p-1)$ matrix $\mathbf{B}$ is full column rank and the vector $\mathbf{b}_{\text{fe}}$ is unknown. Clearly the flat ellipsoidal uncertainty set can be transformed to the model of (11) with $\mathbf{H}_2 = \begin{bmatrix} \mathbf{a}_0 & \mathbf{B} \end{bmatrix}$.

B. Extension of the SSP Method to Multi-Dimension

Firstly let us look at the problem stated as follows: find a vector $\mathbf{a}_{\text{SSP}}$ lying in the signal subspace such that it has the maximum correlation with the nominal manifold vector $\mathbf{a}_0$. By denoting $\mathbf{a}_{\text{SSP}}$ as $\mathbf{a}_{\text{SSP}} = \mathbf{E}_s \mathbf{b}_s$, this problem can be expressed as

$$\rho_s = \max_{\mathbf{b}_s} \frac{(\mathbf{E}_s \mathbf{b}_s)^H \mathbf{a}_0}{\|\mathbf{E}_s \mathbf{b}_s\| \, \|\mathbf{a}_0\|} \tag{14}$$

where ρ_s denotes the maximum correlation between $\mathbf{a}_{\text{SSP}}$ and $\mathbf{a}_0$. In this paper, we assume that the norm of manifold vectors satisfies $\|\mathbf{a}\|^2 = N$. Thus the optimization problem in (14) is equivalent to

$$\max_{\mathbf{b}_s} \ \mathbf{b}_s^H \mathbf{E}_s^H \mathbf{a}_0 \quad \text{s.t. } \mathbf{b}_s^H \mathbf{b}_s = N \tag{15}$$

where $\mathbf{b}_s^H \mathbf{b}_s = \mathbf{b}_s^H \mathbf{E}_s^H \mathbf{E}_s \mathbf{b}_s = \|\mathbf{E}_s \mathbf{b}_s\|^2 = N$. Then the corresponding Lagrangian is

$$L(\lambda, \mathbf{b}_s) = \mathbf{b}_s^H \mathbf{E}_s^H \mathbf{a}_0 - \frac{\lambda}{2}(\mathbf{b}_s^H \mathbf{b}_s - N) \tag{16}$$

Differentiating (16) with respect to $\mathbf{b}_s$ and setting the result to zero produces

$$\mathbf{b}_s = \frac{1}{\lambda} \mathbf{E}_s^H \mathbf{a}_0 \tag{17}$$

where λ can be determined by the constraint $\mathbf{b}_s^H \mathbf{b}_s = N$. Thus the optimal solution is

$$\mathbf{a}_{\text{SSP}} = \mathbf{E}_s \mathbf{b}_s = \frac{1}{\lambda} \mathbf{E}_s \mathbf{E}_s^H \mathbf{a}_0 = \frac{1}{\lambda} \mathbf{P}_{\mathbf{E}_s} \mathbf{a}_0 \tag{18}$$

Replacing $\mathbf{a}_d$ in (7) with $\mathbf{a}_{\text{SSP}}$ above, we can obtain (10). This implies that the SSP method can be derived from the perspective of the problem of maximizing correlation.

Now we consider the following CCA problem: find two vectors located in the subspaces $\langle \mathbf{H} \rangle$ and $\langle \mathbf{E}_s \rangle$ respectively such that the correlation between them is maximized. Let the vector belonging to $\langle \mathbf{H} \rangle$ be represented by $\mathbf{a}_h = \mathbf{Q}_h \mathbf{b}_h$, where $\mathbf{Q}_h \in \mathcal{C}^{N \times p}$ is the orthonormal base for $\langle \mathbf{H} \rangle$ (i.e., $\mathbf{Q}_h^H \mathbf{Q}_h = \mathbf{I}_p$) and $\mathbf{b}_h \in \mathcal{C}^{p \times 1}$ is the combination vector. The matrix $\mathbf{Q}_h$

can be obtained by QR decomposition of $\mathbf{H}$ [1]. Thus the above CCA problem can be expressed as

$$\rho_{sh} = \max_{\mathbf{b}_s,\mathbf{b}_h} \frac{(\mathbf{E}_s\mathbf{b}_s)^H \mathbf{Q}_h\mathbf{b}_h}{\|\mathbf{E}_s\mathbf{b}_s\| \, \|\mathbf{Q}_h\mathbf{b}_h\|} \tag{19}$$

where ρ_{sh} denotes the maximum correlation between two vectors located in $\langle\mathbf{H}\rangle$ and $\langle\mathbf{E}_s\rangle$ respectively. Following the work of [8], the optimization problem in (19) is equivalent to maximizing the numerator subject to keeping the denominator unity, meaning that (19) can be transformed to

$$\begin{aligned}\max_{\mathbf{b}_s,\mathbf{b}_h} \mathbf{b}_s^H\mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h \quad \text{s.t. } \mathbf{b}_s^H\mathbf{b}_s &= N \\ \mathbf{b}_h^H\mathbf{b}_h &= N\end{aligned} \tag{20}$$

The associated Lagrangian is

$$\begin{aligned}L(\lambda_s,\lambda_h,\mathbf{b}_s,\mathbf{b}_h) &= \mathbf{b}_s^H\mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h \\ &\quad -\frac{\lambda_s}{2}(\mathbf{b}_s^H\mathbf{b}_s - N) - \frac{\lambda_h}{2}(\mathbf{b}_h^H\mathbf{b}_h - N)\end{aligned} \tag{21}$$

Taking derivatives with respect to $\mathbf{b}_s$ and $\mathbf{b}_h$ respectively and equating them to zero, we have

$$\frac{\partial L}{\partial \mathbf{b}_s} = \mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h - \lambda_s\mathbf{b}_s = \mathbf{0} \tag{22a}$$

$$\frac{\partial L}{\partial \mathbf{b}_h} = \mathbf{Q}_h^H\mathbf{E}_s\mathbf{b}_s - \lambda_h\mathbf{b}_h = \mathbf{0} \tag{22b}$$

Subtracting $\mathbf{b}_h^H$ times (22b) from $\mathbf{b}_s^H$ times (22a), it is found

$$\begin{aligned}&\mathbf{b}_s^H\mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h - \lambda_s\mathbf{b}_s^H\mathbf{b}_s - \mathbf{b}_h^H\mathbf{Q}_h^H\mathbf{E}_s\mathbf{b}_s + \lambda_h\mathbf{b}_h^H\mathbf{b}_h = \mathbf{0} \\ &\Rightarrow \lambda_h\mathbf{b}_h^H\mathbf{b}_h = \lambda_s\mathbf{b}_s^H\mathbf{b}_s \\ &\Rightarrow \lambda_h = \lambda_s\end{aligned} \tag{23}$$

Also, from (22a) we have

$$\mathbf{b}_s = \frac{\mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h}{\lambda_s} \tag{24}$$

Substituting (24) into (22b) gives

$$\mathbf{Q}_h^H\mathbf{E}_s\mathbf{E}_s^H\mathbf{Q}_h\mathbf{b}_h = \lambda_s^2\mathbf{b}_h \tag{25}$$

Left multiplying the above with $\mathbf{Q}_h$ at both sides produces

$$\mathbf{P}_{\mathbf{H}}\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_h = \lambda_s^2\mathbf{a}_h \tag{26}$$

where $\mathbf{P}_{\mathbf{H}} = \mathbf{Q}_h\mathbf{Q}_h^H$, $\mathbf{P}_{\mathbf{E}_s} = \mathbf{E}_s\mathbf{E}_s^H$, and $\mathbf{a}_h = \mathbf{Q}_h\mathbf{b}_h$. Importantly, (26) tells us that $\mathbf{a}_h$ is corresponding to an eigenvector of the matrix product $\mathbf{P}_{\mathbf{H}}\mathbf{P}_{\mathbf{E}_s}$. Hence the proposed estimated manifold vector is the vector lying in $\langle\mathbf{H}\rangle$ along which the canonical correlation with $\langle\mathbf{E}_s\rangle$ is maximized, i.e.,

$$\widehat{\mathbf{a}}_h = \alpha\mathcal{P}\{\mathbf{P}_{\mathbf{H}}\mathbf{P}_{\mathbf{E}_s}\} \tag{27}$$

where $\mathcal{P}\{\cdot\}$ represents the principal eigenvector of the matrix within the braces, and the constant α is chosen such that $\|\mathbf{a}_h\|^2 = N$.

It is easy to understand that the vector which lies in $\langle\mathbf{E}_s\rangle$ and has maximum canonical correlation with $\langle\mathbf{H}\rangle$ can be given by

$$\mathbf{a}_s = \alpha\mathcal{P}\{\mathbf{P}_{\mathbf{E}_s}\mathbf{P}_{\mathbf{H}}\} \tag{28}$$

If the $N \times p$ matrix $\mathbf{H}$ reduces to one column $\mathbf{a}_0$, then (28) may be rewritten as

$$\mathbf{a}_s^{(1)} = \alpha\mathcal{P}\{\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0\mathbf{a}_0^H\} \tag{29}$$

In addition, we have

$$\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0\mathbf{a}_0^H(\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0) = \mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0\underbrace{(\mathbf{a}_0^H\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0)}_{\text{scalar}} = (\mathbf{a}_0^H\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0)\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0 \tag{30}$$

The above equation shows that $\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0$ is equal to the principal eigenvector of the rank-1 matrix $\mathbf{P}_{\mathbf{E}_s}\mathbf{a}_0\mathbf{a}_0^H$ up to a scaling factor and hence $\mathbf{a}_s^{(1)}$ reduces to $\mathbf{a}_{\text{SSP}}$ in (18). Therefore our proposed estimation algorithm can be viewed as the extension of the SSP method from one dimension to p-dimension.

The proposed beamformer aims to maximize the array output SINR, with the weight vector given by

$$\widehat{\mathbf{w}}_{prop} = \beta\widehat{\mathbf{R}}^{-1}\mathcal{P}\{\mathbf{P}_{\mathbf{H}}\mathbf{P}_{\widehat{\mathbf{E}}_s}\} \tag{31}$$

III. Simulation Results

In order to evaluate the effectiveness of the proposed matched subspace beamforming, three simulation studies have been carried out using, without any loss of generality, a uniform linear array with $N = 10$ sensors placed along the x-axis and half-wavelength sensor spacing. The array operates in the presence of three equally-powered uncorrelated source signals where one is the desired signal and two are interferences. The input SNR is 10dB. The DOAs of the desired signal the first interference signal are, respectively, fixed at $90°$ and $110°$, which are measured anticlockwise from the x-axis. Three beamformers are simulated: the SSP beamformer, Capon beamformer, and the proposed one. The weight vector of Capon beamformer is obtained by using the nominal $\mathbf{a}_0$ instead of $\mathbf{a}_d$ in (7). Also, the optimal SINR curve is plotted in all the following figures for reference, which is computed by using the optimal weight vector in (6). Note that this is impractical because the exact knowledge of $\mathbf{R}_{i+n}$ and $\mathbf{a}_d$ is used.

In the first example, the nominal DOA of the desired signal, θ_0, changes from $85°$ to $95°$ by the step of $1°$. The second interference signal is from the direction of $75°$. The matrix $\mathbf{H}$ is composed of the nominal manifold and the first two derivatives with respect to the nominal DOA. Fig.1 shows the array output SINR versus the look direction errors, where we can see that our proposed method gains higher SINR than the SSP method except the case where $\theta_0 = 95°$. The performance of Capon method is the worst because the traditional Capon beamformer does nothing to achieve the robustness.

The nominal DOA θ_0 is fixed at $87°$ (i.e., $-3°$ look direction error) in the second example. The DOA of the second interference signal varies from $75°$ to $85°$. The remaining parameters are the same as that in the first example. Fig.2 illustrates the output SINR against the direction of the second interference signal. To degrade the performance of the proposed method, the DOA of the second interference is needed to be closer to θ_d than that for the SSP method.

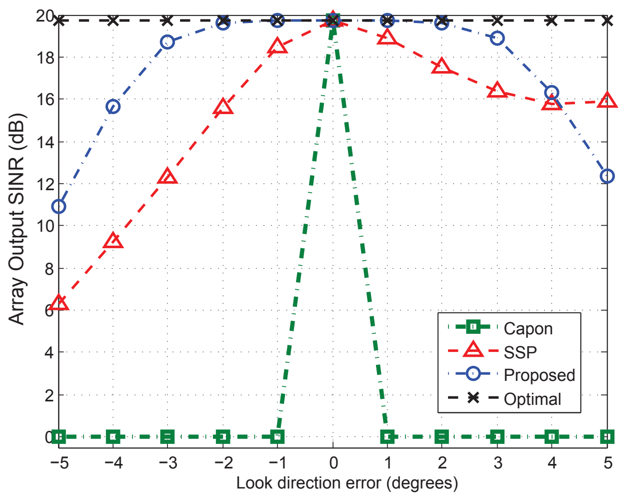

Fig. 1. Array output SINR versus look direction error; the first example.

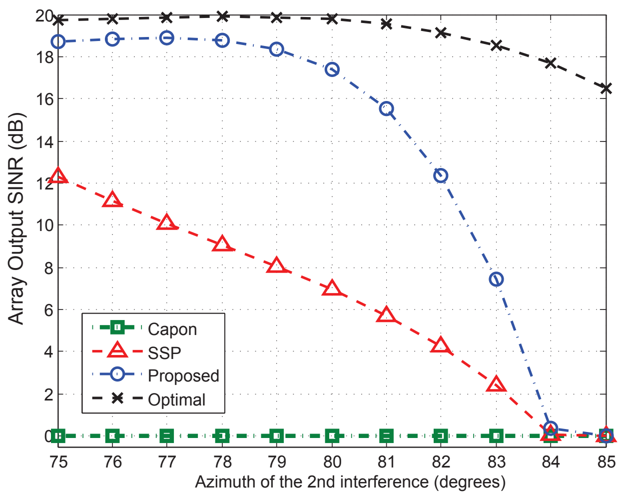

Fig. 2. Array output SINR versus the azimuth of the second interference; the second example.

In the final example, we test the effect of finite snapshots where the covariance matrix is formed by using (2). The parameters here are the same as that in the second example except the DOA of the second interference signal fixing at $75°$. Fig. 3 displays the SINR versus the available snapshot number where the averages of 500 independent Monte-Carlo runs are used to plot each simulation point. We can see that when the snapshot number is relatively large (over 150 in this example) the proposed method outperforms the SSP method in terms of array output SINR.

IV. Conclusion

Under the assumption that the manifold vector of the desired signal belongs to a known linear subspace, we extend the SSP method from one dimension to multiple dimension by means of the CCA technique. The proposed estimated manifold is

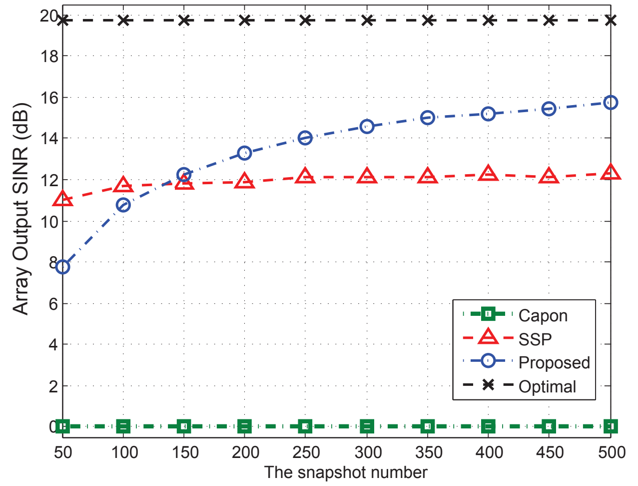

Fig. 3. Array output SINR versus the snapshot number; the fourth example.

scaled to the principal eigenvector of the product of the signal-subspace projector and the linear subspace projector. The simulations reveal that our proposed method can achieve better performance than the SSP approach. A future area of research consists of improving the proposed method so that it gains better performance in the case of quite few snapshots, say, the order of array sensor number.

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