Low Complexity 2-D DOA Estimator for Arbitrary Arrays: a Hybrid MUSIC-based Method

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***Abstract*—The traditional search-based MUltiple SIgnal Classification (MUSIC) method is often computationally expensive, particulary for the application of joint azimuth and elevation estimation. By means of the manifold separation technique (MST), the search-free root-MUSIC method, which is originally designed for the uniform linear array structures, can be extended to arbitrary arrays and reduce the computation burden to some extent. However, a computationally complex polynomial rooting procedure is still required. In this paper, we propose a computation attractive 2-D direction-of-arrival (DOA) estimator which can be viewed as a hybrid MUSIC-based method. First, we use the MST method to convert the 2D-MUSIC cost function into stand 2D-IDFT form. In doing so, we can obtain the 2-D spatial spectrum by using 2D-FFT. Since a relatively small point number for the FFT is chosen, the DOAs are located roughly. Then the MUSIC method with fine angular grid is utilized to search the DOAs finely within a small angular section. The proposed hybrid method not only alleviates the computation burden of root-MUSIC or MUSIC solely used; it also achieves almost the same DOA estimation performance and is easy to implement.**

***Keywords*—2-D DOA estimation; MUSIC; root-MUSIC; arbitrary arrays.**

I. Introduction

Direction-of-arrival (DOA) estimation is a ubiquitous task concerned in array processing. The well-known MUltiple SIgnal Classification (MUSIC) method, a classical subspace-type method, asymptotically exhibits infinite resolution capabilities and thus are classified as a super-resolution technique in estimating DOA. The major drawback associated with MUSIC is that the computational complexity is high, particularly for the real-time applications. In order to alleviate the computational burden, the root-MUSIC algorithm converts the DOA estimation to a problem of polynomial rooting by exploiting the Vandermonde structured array manifold vector of uniform linear arrays (ULA). In [1], [2] the conventional root-MUSIC has been extended to arrays with arbitrary geometry by using the manifold separation technique (MST). An alternative technique, called Fourier-domain root-MUSIC proposed in [3], can also extend the root-MUSIC to arbitrary arrays. In [4], [5] two improved methods are presented to reduce the computational burden of the extended root-MUSIC. Nevertheless, only azimuth estimation is considered in the above works.

In [6], joint azimuth and elevation estimation for arbitrary arrays is concerned. Also, the MST technique is used in [6] to convert the 2-D DOA estimation to a problem of bivariate polynomial rooting. However, if the array aperture becomes large, the order of the the bivariate polynomial must be quite large which leads to unacceptable computational complexity for the rooting procedure. In this paper, we propose a computation attractive 2-D DOA estimator. First, we convert the 2D-MUSIC cost function into stand 2D-IDFT form by using the MST technique. In doing so, we can obtain the 2-D spatial spectrum by using 2D-FFT. For the sake of computation efficiency, a relatively small point number for the FFT is chosen and thus the DOAs are located roughly. Then, the MUSIC method with fine angular grid is utilized to search the DOAs precisely within a small angular section. The proposed hybrid method not only alleviates the computation burden of root-MUSIC or MUSIC solely used; it also achieves the same DOA estimation performance and is easy to implement.

II. Signal Model

Consider a non-ULA array of N sensors operating in the presence of M uncorrelated narrowband signals via unknown directions. The $N \times 1$ array receiver vector can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{m}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{n}(t)$ represents the additive white Gaussian noise with covariance $\sigma_n^2 \mathbf{I}_N$ (σ_n^2 is the noise power). The matrix $\mathbf{A}$ collects the manifold vectors of the M signals, i.e.,

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1),\ \mathbf{a}(\theta_2, \phi_2),\ ...,\ \mathbf{a}(\theta_M, \phi_M)] \tag{2}$$

where $\theta \in [0, 2\pi)$ and $\phi \in [0, \pi]$ denote the azimuth and elevation angle respectively. The vector $\mathbf{m}(t)$ has elements the M complex narrowband signal envelopes. In practical applications, the second order statistics of $\mathbf{x}(t)$ can be obtained as follows

$$\widehat{\mathbf{R}} = \frac{1}{K}\sum_{l=1}^{K} \mathbf{x}(t_k)\mathbf{x}^H(t_k) \tag{3}$$

where $\{\mathbf{x}(t_k)\}_{k=1}^{K}$ denote the K received snapshots. Then performing eigenvalue decomposition on $\widehat{\mathbf{R}}$ produces

$$\widehat{\mathbf{R}} = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H \tag{4}$$

where $\mathbf{E}_s \in \mathcal{C}^{N \times M}$ is the eigenvectors associated with the largest M eigenvalues and $\mathbf{E}_n \in \mathcal{C}^{N \times (N-M)}$ represents the eigenvectors corresponding to the remaining small eigenvalues. Commonly $\mathbf{E}_s$ and $\mathbf{E}_n$ are referred to as the signal-subspace eigenvectors and noise-subspace eigenvectors. The diagonal

matrices $\mathbf{\Lambda}_s$ and $\mathbf{\Lambda}_n$ have diagonal elements associated with the signal and noise eigenvalues respectively.

Due to the orthogonality between the signal and noise subspace, the angles where the projection of the corresponding manifold onto the noise subspace are zero should be the DOAs of signals. By exploiting this property, the MUSIC searches the continuous array manifold vector over the area of θ and ϕ to find the M minima of the following null-spectrum cost function

$$(\widehat{\theta}, \widehat{\phi}) = \arg\min_{\theta,\phi}\{\mathbf{a}(\theta,\phi)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta,\phi)\}. \tag{5}$$

However, the spectral search process required by MUSIC may be unaffordable for some real-time implementations, which can be explained by the following fact. To obtain each search point, a matrix product of $\mathbf{a}(\theta,\phi)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta,\phi)$ has to be computed. Moreover, for the purpose of avoiding grid error, the required number of search points has to be significantly large. In other words, the angular search grid has to fine enough to avoid the quantization problem. This drawback becomes particularly apparent in joint estimation of azimuth and elevation since we have to search over two dimensions.

III. Manifold Separation Technique with 2-D Wavefield

Interestingly, by using the manifold separation technique (MST) [1], [2], [7], a $(N \times 1)$ manifold vector can be modeled as

$$\mathbf{a}(\theta,\phi) = \mathbf{\Gamma}\mathbf{d}(\theta,\phi) + \varepsilon \tag{6}$$

where ε denotes the modeling error. The vector $\mathbf{d}(\theta,\phi)$ consists of the following Vandermonde structured vectors

$$\mathbf{d}(\theta,\phi) = \mathbf{d}(\theta) \otimes \mathbf{d}(\phi) \in \mathcal{C}^{\mathcal{M}_a \mathcal{M}_e \times 1} \tag{7}$$

where

$$\begin{aligned} \mathbf{d}(\theta) &= \frac{1}{\sqrt{2\pi}}[z_\theta^{\frac{\mathcal{M}_a-1}{2}}, \cdots, 1, \cdots, z_\theta^{-\frac{\mathcal{M}_a-1}{2}}]^T \in \mathcal{C}^{\mathcal{M}_a \times 1} \\ \mathbf{d}(\phi) &= \frac{1}{\sqrt{2\pi}}[z_\phi^{\frac{\mathcal{M}_e-1}{2}}, \cdots, 1, \cdots, z_\phi^{-\frac{\mathcal{M}_e-1}{2}}]^T \in \mathcal{C}^{\mathcal{M}_e \times 1} \end{aligned} \tag{8}$$

with $z_\theta = e^{j\theta}$, $z_\phi = e^{j\phi}$ and $\otimes$ representing the Kronecker product. We can see that the vector $\mathbf{d}(\theta,\phi)$ depends only on the wavefield. The matrix $\mathbf{\Gamma}$ is called the sampling matrix given by

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{g}_1^T \\ \vdots \\ \mathbf{g}_N^T \end{bmatrix} \in \mathcal{C}^{N \times \mathcal{M}_a \mathcal{M}_e} \tag{9}$$

which depends only on the sensor array configuration and its properties. Let us have a close look at the n-th element of the manifold vector $\mathbf{a}(\theta,\phi)$ (i.e., the array response of the n-th sensor) which is given by

$$\begin{aligned} [\mathbf{a}(\theta,\phi)]_n &= \mathbf{g}_n^T \mathbf{d}(\theta,\phi) \\ &= (\mathbf{d}(\theta) \otimes \mathbf{d}(\phi))^T \mathbf{g}_n \\ &= \mathrm{vec}\{(\mathbf{d}^T(\theta) \otimes \mathbf{d}^T(\phi))\mathbf{g}_n\} \\ &= \mathrm{vec}\{\mathbf{d}^T(\phi)\mathbf{G}_n\mathbf{d}(\theta)\} \\ &= \mathbf{d}^T(\phi)\mathbf{G}_n\mathbf{d}(\theta) \end{aligned} \tag{10}$$

where $\mathbf{g}_n = \mathrm{vec}\{\mathbf{G}_n\}$. The notation $\mathrm{vec}\{\cdot\}$ denotes the vectorization operator which stacks the columns of a matrix on top of each other. The properties $(\mathbf{X} \otimes \mathbf{Y})^T = \mathbf{X}^T \otimes \mathbf{Y}^T$ and $\mathrm{vec}\{\mathbf{XYZ}\} = (\mathbf{Z}^T \otimes \mathbf{X})\mathrm{vec}\{\mathbf{Y}\}$ are used in the above. From (10), it is clear that the essence of the MST model in (6) is using a number of 2-D Fourier series to approximate the array response. Specifically, the model error ϵ can be safely neglected as the mode numbers $\mathcal{M}_a$ and $\mathcal{M}_e$ are both large enough.

In practical applications, the real-world array manifold vectors are often not well described by their theoretical models and there exist mismatches between them, which may be caused by many factors, e.g., mutual coupling, physical location misplacement of array elements and mounting platform reflections. Unfortunately, these mismatches are usually unknown explicitly. In order to address this problem, array calibration works are needed. In this paper, we employ the so-called 2-D Effective Aperture Distribution Function (EADF) [7] to estimate the sampling matrix from calibration measurements.

The aim of estimating the sampling matrix is twofold. First, the sampling matrix can accommodate the array nonidealities and hence the model in (6) is suitable for real-world arrays. Second, we can exploit the Vandermonde structure in (6) to apply computational efficient DOA estimators to arbitrary arrays. We can acquire the sampling matrix for the real-world arrays through measurements from a number of different location of angles. Typically, these calibration measurements are obtained in controlled environments such as anechoic chambers. The antenna array is mounted on a mechanical platform and then be rotated in different azimuths and elevations, while a known active source is held fixed. Then we perform 2-D IDFT on these measurements to compute the sampling matrix. See [6], [7] for more details. In this paper, we assume that the sampling matrix has been obtained off-line prior to DOA estimations and we place focus on the DOA estimations.

IV. Proposed Hybrid MUSIC-based Method

Now, inserting the MST expansion model in (6) into the MUSIC cost function in (5), we have the spatial spectrum

$$\begin{aligned} p(\theta,\phi) &= \mathbf{d}(\theta,\phi)^H(\mathbf{\Gamma}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{\Gamma})\mathbf{d}(\theta,\phi) \\ &= \mathbf{p}(\theta)^T\mathbf{C}\mathbf{p}(\phi) \end{aligned} \tag{11}$$

where $\mathbf{p}(\theta) = [z_\theta^{2\mathcal{M}_a-2}, z_\theta^{2\mathcal{M}_a-3}, \ldots, 1]^T$ and $\mathbf{p}(\phi) = [z_\phi^{2\mathcal{M}_e-2}, z_\theta^{2\mathcal{M}_e-3}, \ldots, 1]^T$ are Vandermonde structured vectors. The matrix $\mathbf{C} \in \mathcal{C}^{(2\mathcal{M}_a-1)\times(2\mathcal{M}_e-1)}$ can be found by the following steps. First, by defining the matrix $\mathbf{B} = \mathbf{\Gamma}^H\mathbf{E}_n\mathbf{E}_n^H\mathbf{\Gamma}$ and expressing it in block form, we have

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{1,1} & \mathbf{B}_{1,2} & \ldots & \mathbf{B}_{1,\mathcal{M}_e} \\ \mathbf{B}_{2,1} & \mathbf{B}_{2,2} & \ldots & \mathbf{B}_{2,\mathcal{M}_e} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{\mathcal{M}_e,1} & \mathbf{B}_{\mathcal{M}_e,2} & \ldots & \mathbf{B}_{\mathcal{M}_e,\mathcal{M}_e} \end{bmatrix} \tag{12}$$

where each block matrix is a $\mathcal{M}_a$-by-$\mathcal{M}_a$ matrix. Then computing the sum of the block elements along all $2\mathcal{M}_e - 1$

diagonals produces

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_2 & \dots & \mathbf{D}_{2\mathcal{M}_e-1} \end{bmatrix} \tag{13}$$

with $\mathbf{D}_i = \sum_{\forall \mathcal{M}_e-(m-n)=i} \mathbf{B}_{m,n}$. Similarly, for the i-th column of $\mathbf{C}$, we compute the sum of the elements along all $2\mathcal{M}_a - 1$ diagonals of $\mathbf{D}_i$, i.e., $[\mathbf{C}]_{i,q} = \sum_{\forall \mathcal{M}_a-(m-n)=q} [\mathbf{D}_i]_{m,n}$.

Substituting $\theta = \frac{2\pi}{N_1} n_1$ and $\phi = \frac{2\pi}{N_2} n_2$ into (11), we can rewrite the spatial spectrum as

$$p(n_1, n_2) = \sum_{k_2=0}^{d_e} \sum_{k_1=0}^{d_a} C(k_1, k_2) e^{j\frac{2\pi}{N_1} n_1 k_1} e^{j\frac{2\pi}{N_2} n_2 k_2} \tag{14}$$

where $d_a = 2\mathcal{M}_a - 2$, $d_e = 2\mathcal{M}_e - 2$ and $C(k_1, k_2) = [\mathbf{C}]_{d_a+1-k_1, d_e+1-k_2}$. The notation $[\mathbf{C}]_{m,n}$ stands for the (m, n)-th element of $\mathbf{C}$. Apparently, the expression in (14) is of typical 2-D IDFT form and therefore we can apply the well-known 2-D FFT algorithm to calculate the 2-D spatial spectrum. In comparison with the 2-D MUSIC method in which each search point requires a matrix computation, the whole search points can be found after the 2-D FFT algorithm is performed. In order to avoid grid errors, the numbers N_1 and N_2 have to take large values, say 4096 to achieve grid of 0.088 degree. Since performing 2-D FFT on the matrix $\mathbf{C}$ requires to compute N_1-point FFT for the $2\mathcal{M}_e - 1$ columns of $\mathbf{C}$ and then N_2-point FFT of N_1 rows, the computational complexity may still be expensive for some real-time applications. Next, in order to reduce the computation burden further, we propose a hybrid method which can be accomplished via the following steps.

1) For the calibration purpose, measure the array response at a number of locations and calculate the sampling matrix $\mathbf{\Gamma}$ via 2-D IDFT. Note that this off-line process needs to be done only once for a given array.
2) Collect the snapshots to form the covariance matrix in (3) and perform the eigen-decomposition to obtain the noise eigenvectors $\mathbf{E}_n$ and the matrix $\mathbf{B} = \mathbf{\Gamma}^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{\Gamma}$.
3) Sum the block elements along the diagonals of $\mathbf{B}$ to obtain the matrix $\mathbf{D}$ and then sum the elements along the diagonals of each block elements in $\mathbf{D}$ to form the matrix $\mathbf{C}$.
4) Employ 2-D FFT algorithm on the matrix $\mathbf{C}$ where N_1 and N_2 can be moderate numbers, say 256. Then search the minima of this coarse spatial spectrum, which corresponds to the DOAs roughly.
5) Within a small angular section centered at each rough DOA, apply the 2-D MUSIC method to locate the DOAs finely. For instance, for a rough DOA (θ_c, ϕ_c), compute the 2-D MUSIC spectrum in (5) where $\theta \in [\theta_c - \Delta\theta, \theta_c + \Delta\theta]$ and $\phi \in [\phi_c - \Delta\phi, \phi_c + \Delta\phi]$ with $\Delta\theta$ and $\Delta\phi$ being the search ranges. In such angular section, fine grid is utilized to avoid grid errors.

Remark: The complexity order of the 2-D FFT of $\mathbf{C}$ is $O((2\mathcal{M}_e - 1)N_1 \log N_1 + N_1 N_2 \log N_2)$. It is clear that if we use moderate N_1 and N_2, the computational complexity can be reduced significantly. Further, the 2-D MUSIC performed within each small angular section has the complexity order of $O(Q_1 Q_2 N)$ where Q_1 and Q_2 are the grid numbers of azimuth and elevation respectively, and N is the antenna number (normally $N \ll N_1$ and $N \ll N_2$). One may wonder whether it is suitable to apply the zoom FFT algorithm to find the local spatial spectrum. However, here the zoom FFT may require more computations than 2-D MUSIC. Since the complexity order the zoom FFT is $O(N_1 \log Q_1 + 2N_1)$ for each 1-D N_1-point zoom FFT with Q_1 output points, the overall complexity of the zoom FFT is $O((2\mathcal{M}_e - 1)(N_1 \log Q_1 + 2N_1) + Q_1(N_2 \log Q_2 + 2N_1))$ where N_1 and N_1 are large numbers because fine grid is needed in the local angular spectrum.

V. Simulation Results

In this section we evaluate the effectiveness of the proposed hybrid MUSIC-based approach. In each simulation run, we randomly generate a Non-ULA antenna array with $N = 5$ sensors and then use the ideal manifold model to compute the sampling matrix with $\mathcal{M}_a = \mathcal{M}_e = 51$. Three methods are simulated: the 2-D root-MUSIC in (14) with $N_1 = N_2 = 256$ and $N_1 = N_2 = 4096$, and the proposed hybrid method. In the hybrid method, we first perform 2-D FFT with $N_1 = N_2 = 256$ to locate the DOAs coarsely. Then at each rough DOA point, we apply 2-D MUSIC with grid of $\frac{360}{4096} \approx 0.088$ degree and angular range $\Delta\theta = \Delta\phi = 3°$. That is the local grid numbers are $Q_1 = Q_2 = 69$. We assume that two signals from different angles impinge the antenna array. Also, the DOAs are randomly generated in each simulation run.

The root-mean-square errors (RMSEs) between the estimated and real DOAs are investigated in Fig. 1 and Fig. 2 with input signal-to-noise ratio (SNR) varying from -10dB to 20dB while the snapshot number is fixed at 200. In Fig. 3 and Fig. 4, we also illustrate the RMSE performance for the DOA estimators versus the snapshot number where the SNR is kept at 10dB. In these four figures, we can see that despite significant computational reduction, the proposed hybrid method can achieve almost the same DOA estimation performance with the full 2-D root-MUSIC with $N_1 = N_2 = 4096$ and outperform the 2-D root-MUSIC with $N_1 = N_2 = 256$.

VI. Conclusion

Joint azimuth and elevation estimation is often a computationally expensive process. In this paper, we present a computation attractive 2-D DOA for arbitrary arrays. By using the MST method, the 2D-MUSIC spatial spectrum can be found by applying the 2D-FFT algorithm. In order to avoid grid errors, the point numbers for the FFT have to be quite large which leads to high computational complexity. To address this problem, we propose a two-step method. First, we perform the 2-D FFT with small point numbers to locate the DOAs coarsely. Then 2-D MUSIC with fine angular grid is utilized to search the DOAs finely within a small angular section. The simulation results show that the proposed hybrid method not only alleviates the computation burden but also achieves almost the same DOA estimation performance.

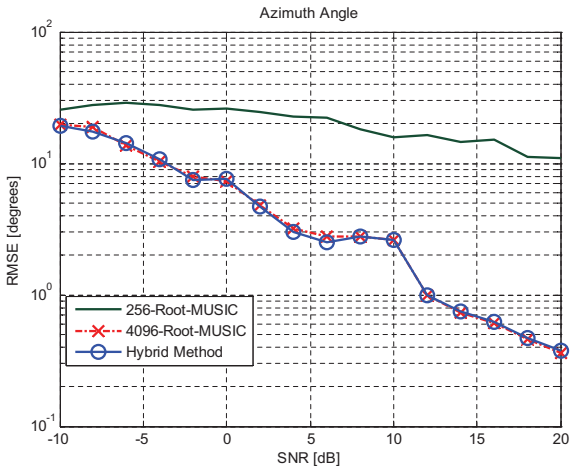

Fig. 1. Azimuth estimation RMSE versus input SNR.

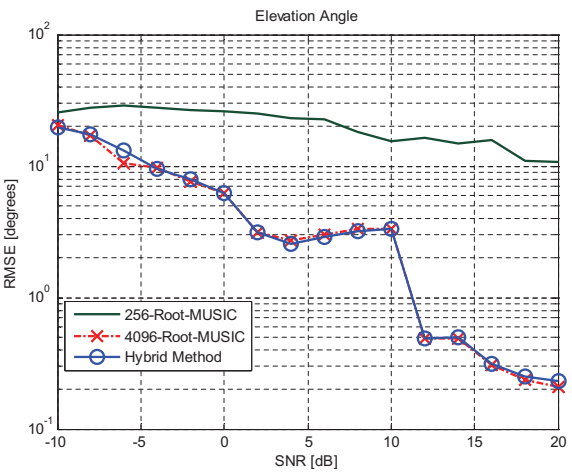

Fig. 2. Elevation estimation RMSE versus input SNR.

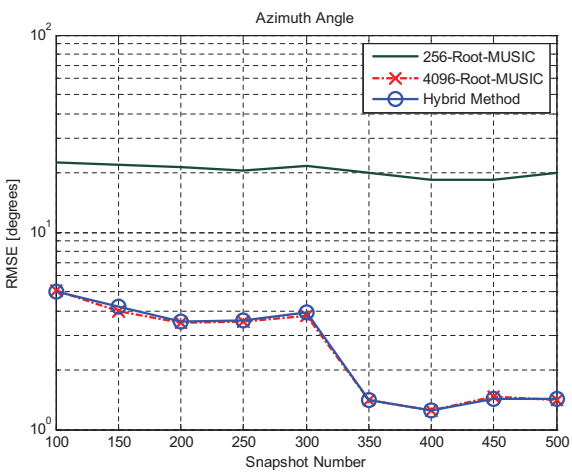

Fig. 3. Azimuth estimation RMSE versus snapshot number.

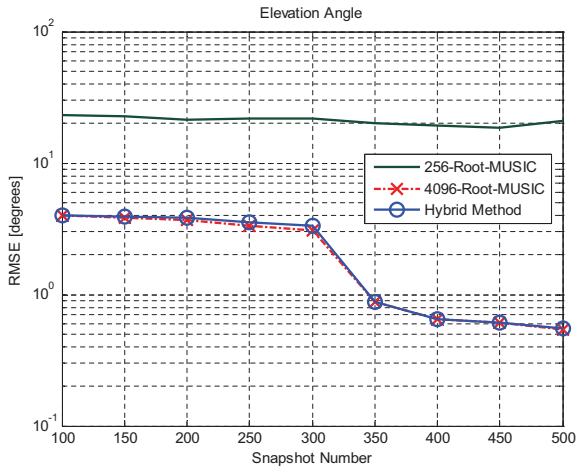

Fig. 4. Elevation estimation RMSE versus snapshot number

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